Femtoscopy in $\sqrt{s}$ = 200 GeV *p*+*p* collisions at RHIC-PHENIX

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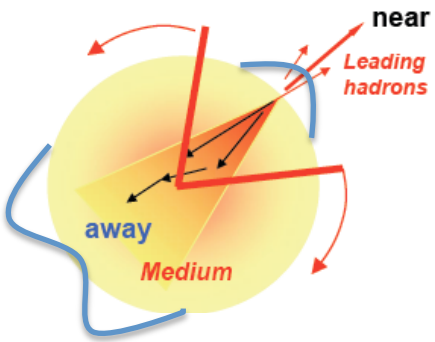
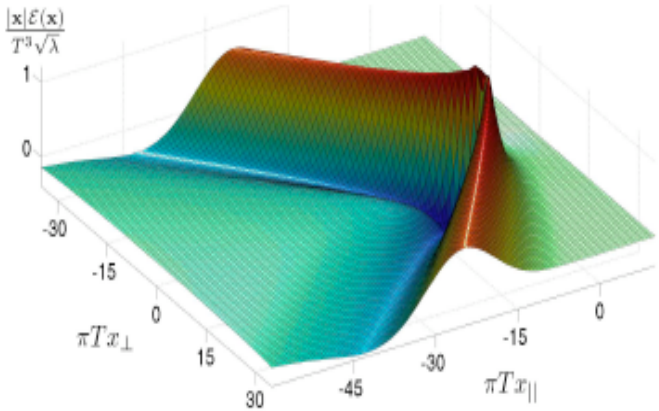
HAW09: Joint APS/DNP JPS Meeting

Why measure Hanbury Brown Twiss in p+p?

- Baseline measurement for heavy ion
- Proving ground for understanding and dealing with non-HBT correlations (energy-momentum correlations)
  - Event selection
  - Relative to jet
  - Multiplicity
  - …
- Capability likely needed for future heavy ion analyses relative to jet axis. Medium response measurable in freeze out hyper-surface? (Similar to HBT relative to reaction plane.)

Z. Chajecki, M. Lisa
Phys.Rev.C78:064903,2008





Chesler & Yaffe, 0706.0368(hep-th)

HBT nomenclature

$$C_2(\mathbf{q}) = A(\mathbf{q})/B(\mathbf{q}) = \frac{\text{Actual q distribution}}{\text{Background q distribution}} \qquad \mathbf{q} = \mathbf{p_1} - \mathbf{p_2}$$

$$= 1 + \int d\mathbf{r} K(\mathbf{q},\mathbf{r}) S(\mathbf{r})$$

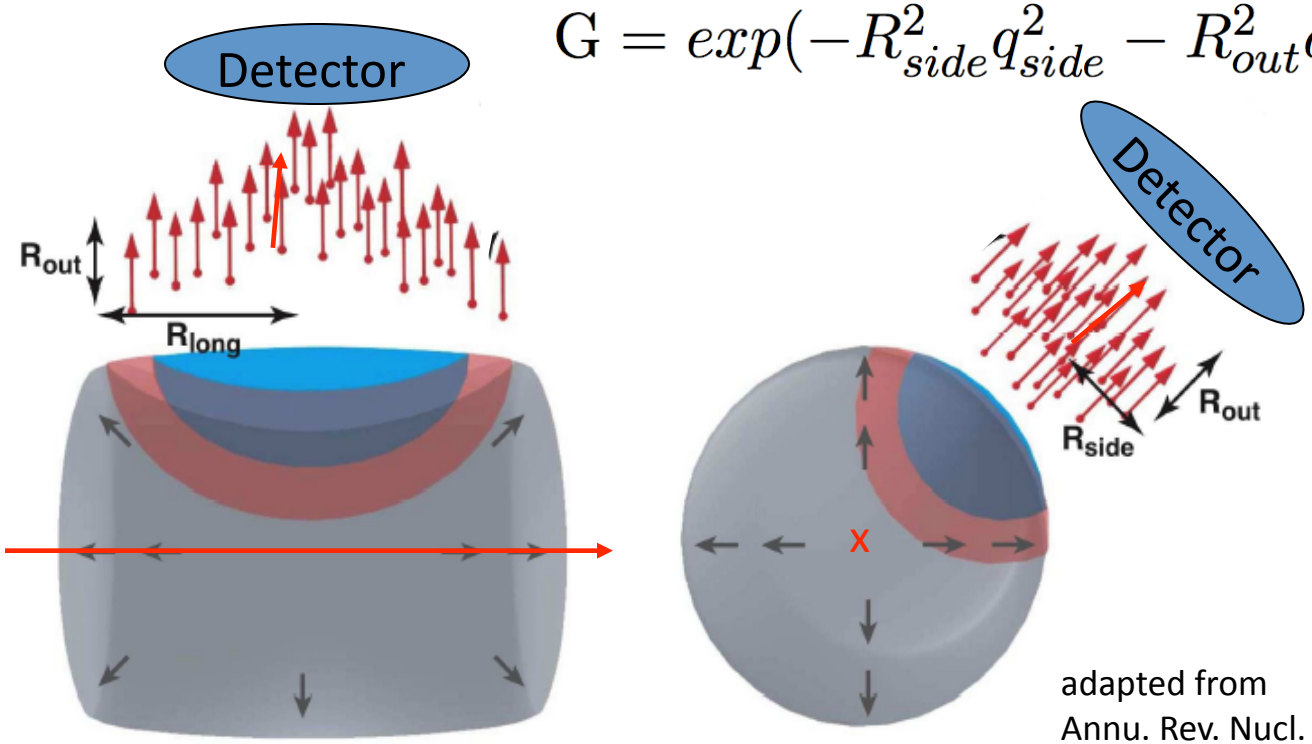
The source *S* can be directly recovered with imaging

or

$$= \lambda(1 + \mathrm{G})\mathrm{F_C} + 1 - \lambda$$

Make assumptions about the source

$$\mathrm{G} = exp(-R^2_{side} q^2_{side} - R^2_{out} q^2_{out} - R^2_{long} q^2_{long})$$



Bose-Einstein Enhancement at Low q

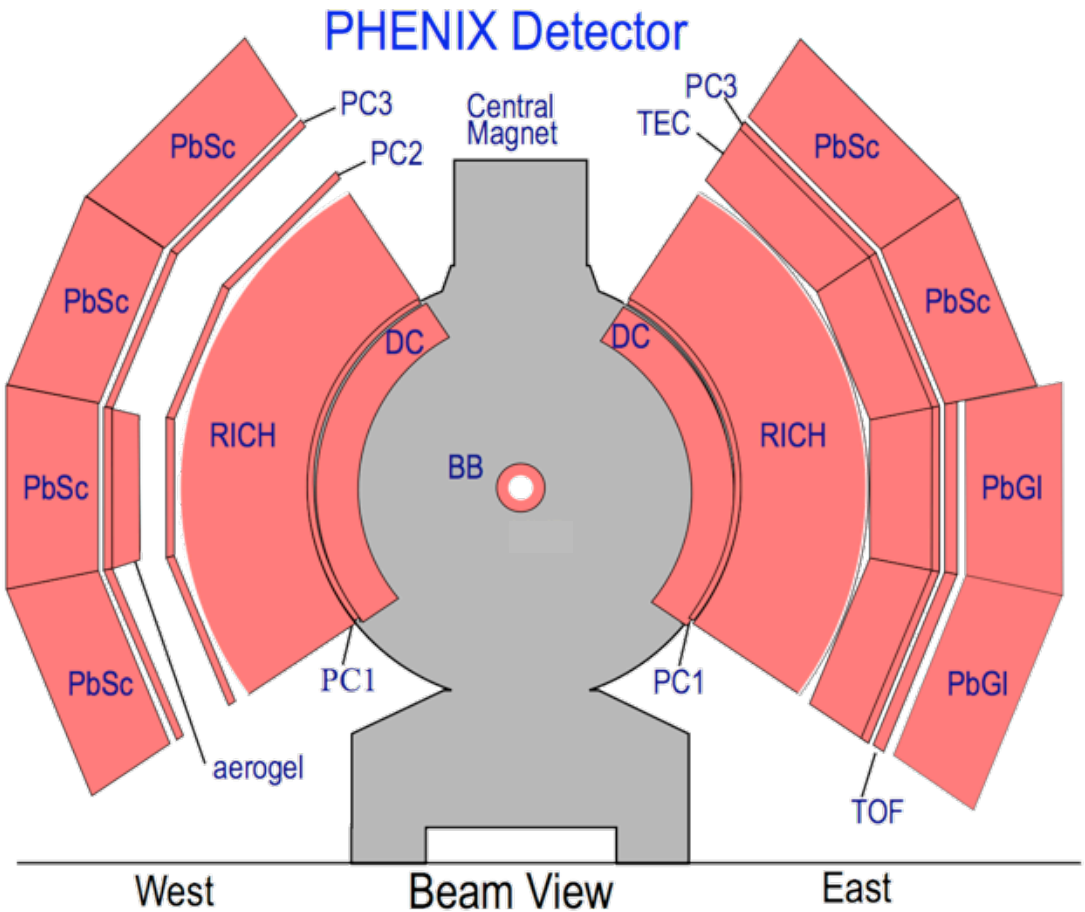
Simplified for identical particles

$$R_{out} \approx R_{side} + v_\perp \Delta\tau$$

$$\tau \approx \frac{R_{long}}{v_{therm}}$$

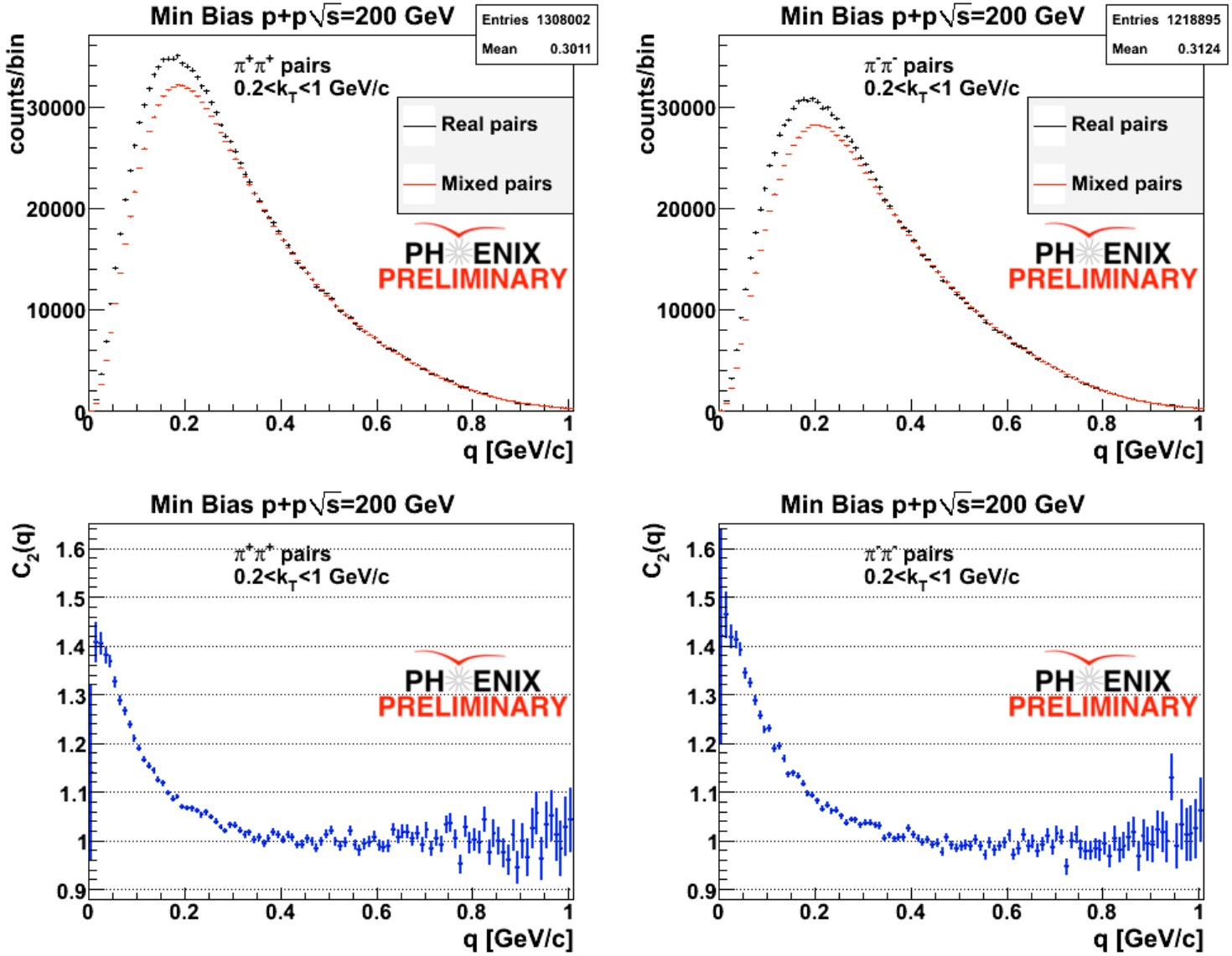
adapted from Annu. Rev. Nucl. Part. Sci. 2005. 55:357–402

A few analysis details



- Run5 p+p minimum biased (~2.5M like sign pion pairs)
- Charged pions identified by time of flight from West PbSc
- Matching cuts reduce backgrounds
- Pair selection cuts to remove merging and splitting
- No Monte Carlo based corrections so more selective pair cuts

1-D Correlations



Staring point for 1D imaging analysis

Slices of 3-D Correlation

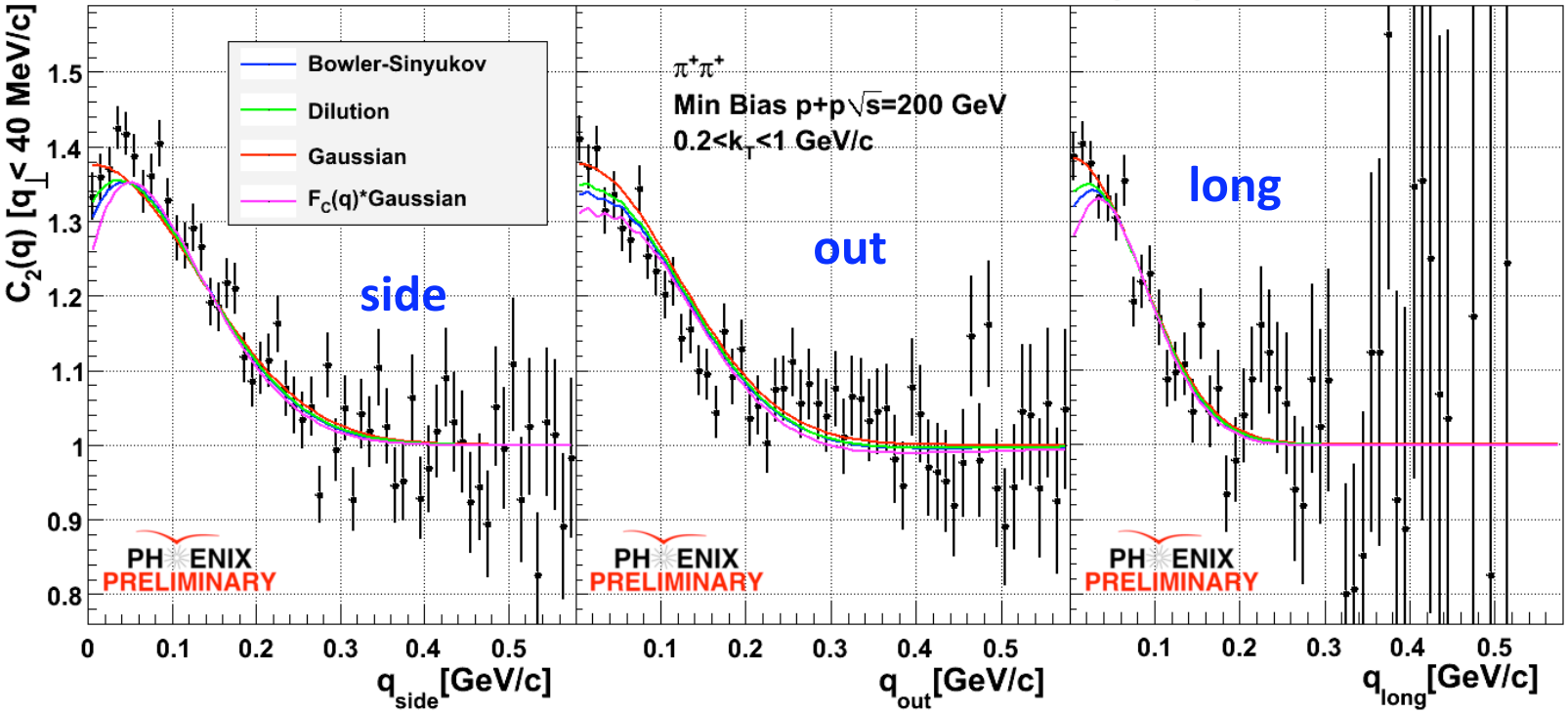
$$\text{Bowler-Sinyukov: } C_2 = [\lambda(1+G)]F_c(q_{inv}) + [1+\lambda]$$

$$\text{Dilution: } C_2 = [\lambda(1+\lambda G)]F_c(q_{inv}) + [1+\lambda]$$

$$\text{Coulomb corrected Gaussian: } C_2 = (1+\lambda G)F_c(q_{inv})$$

$$\text{Gaussian: } C_2 = 1+\lambda G$$

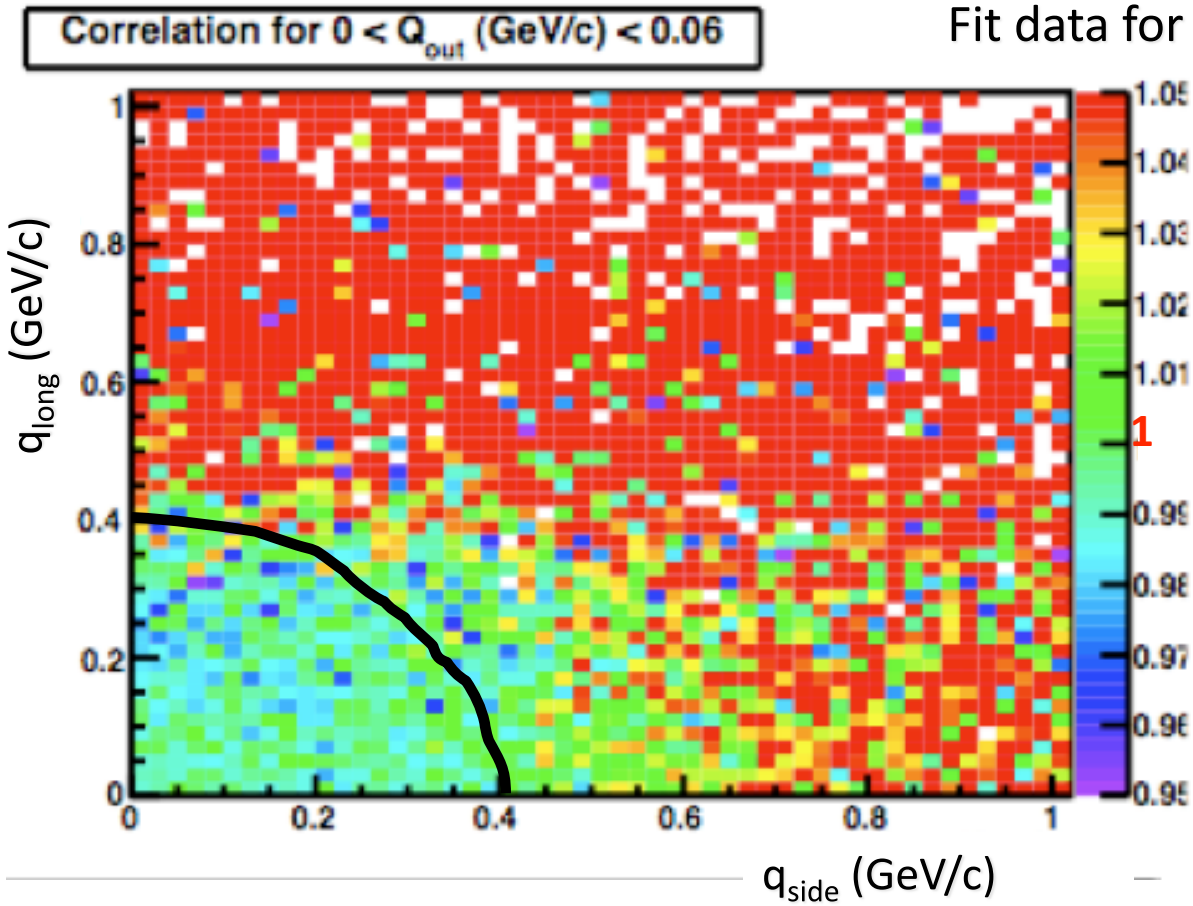
$$\text{where } G = \exp(R_{side}^2 q_{side}^2 + R_{out}^2 q_{out}^2 + R_{long}^2 q_{long}^2)$$



A few words on fitting

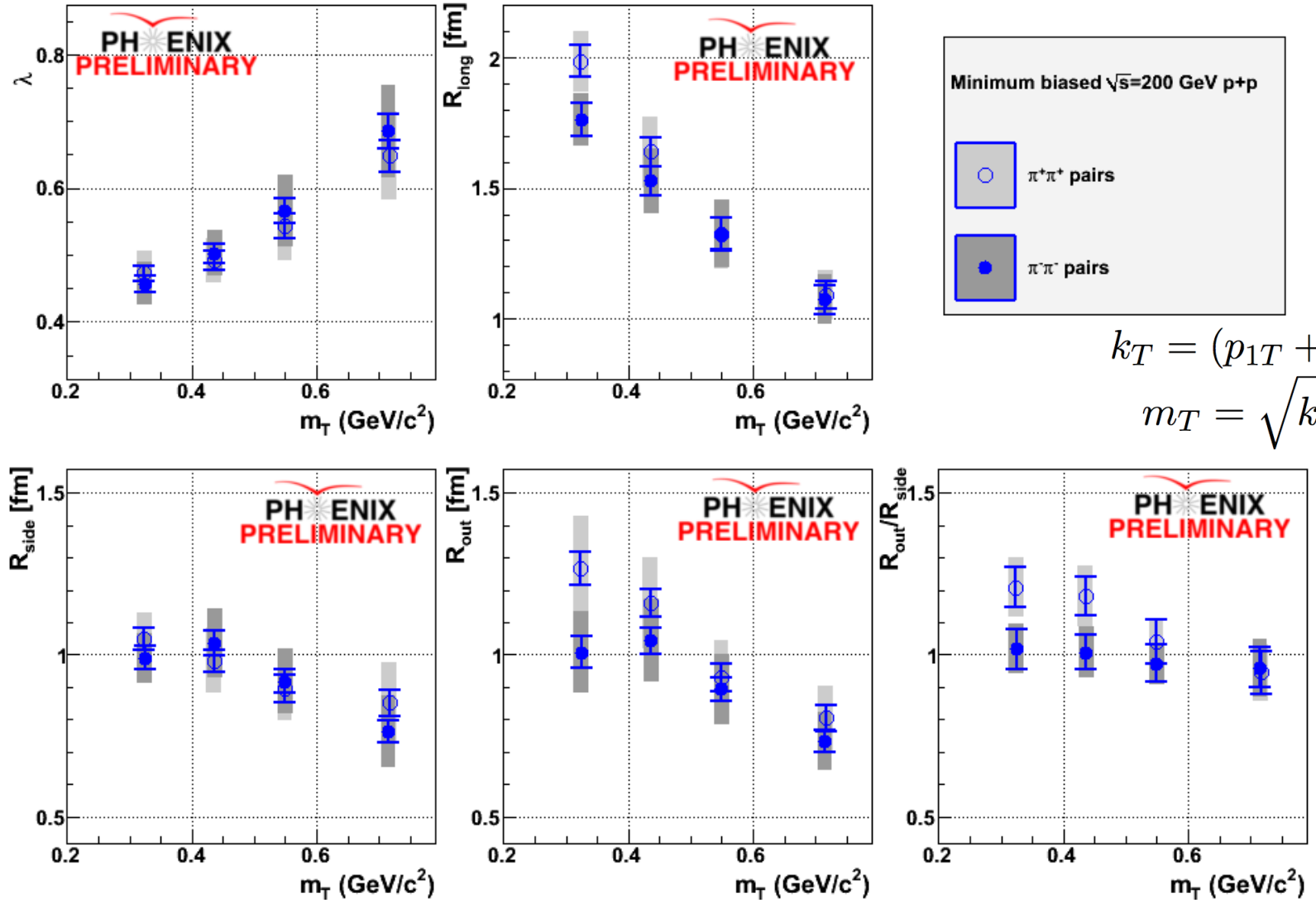
PYTHIA used as sanity check to limit fit range
and help minimize impact of non-HBT correlations

Fit data for $q_o^2+q_l^2+q_s^2<(0.4\text{GeV/c})^2$



one example of
many slices

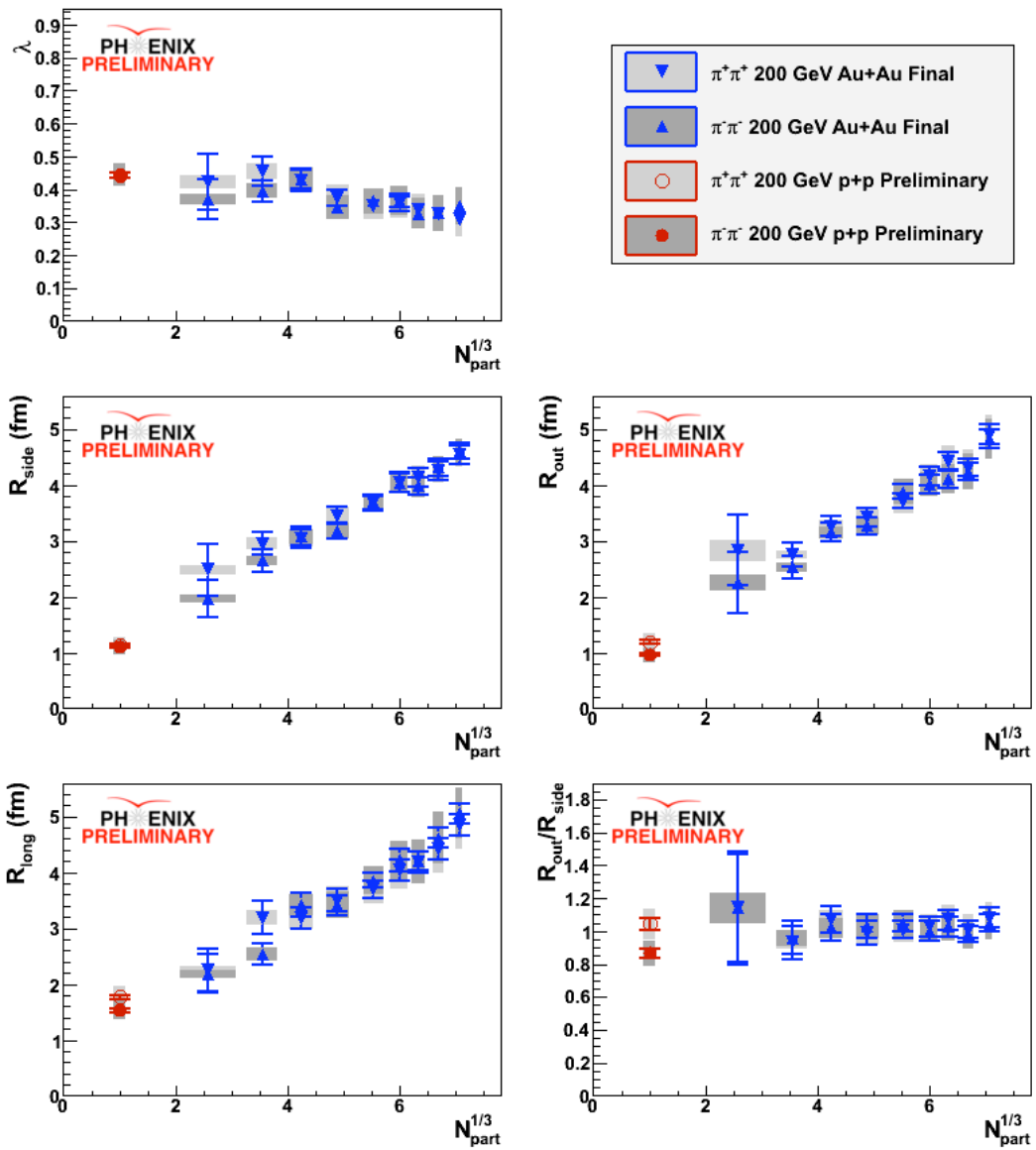
Transverse Mass Dependence



$$k_T = (p_{1T} + p_{2T})/2$$

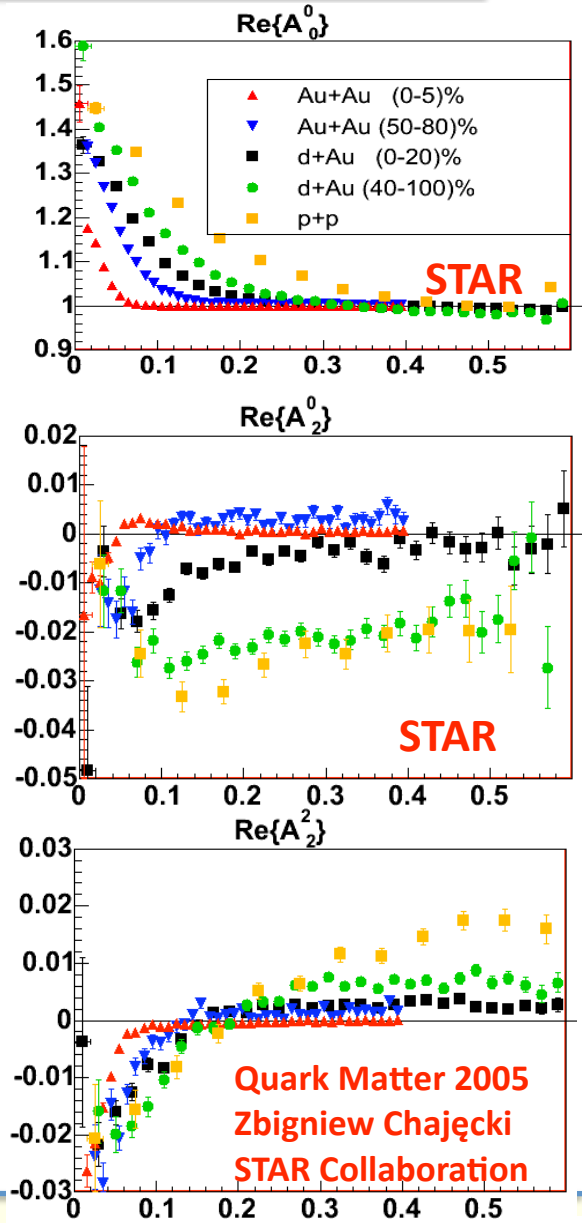
$$m_T = \sqrt{k_T^2 + m^2}$$

Au+Au Comparison



Spherical Harmonic Expansion

- Preferred for imaging over spline basis
  - D. A. Brown and P. Danielewicz, Phys. Lett. B 398, 252 (1997) [arXiv:nucl-th9701010]
  - D. A. Brown *et. al.* Phys.Rev.C72:054902,2005.
- May reveal non-HBT correlations in a more immediate way.
- Traditionally starts from 3D foreground and background distributions
  - Maximizes statistics and acceptance issues
  - New method for directly computing A. Kisiel and D. Brown arXiv:0901.3527



Direct Spherical Harmonic Decomposition

$$T(\mathbf{q}) = C(\mathbf{q})M(\mathbf{q})$$

$$T(\mathbf{q}) = \sqrt{4\pi}\sum_{\ell m} T_{\ell m}(q)Y_{\ell m}(\Omega_{\hat{\mathbf{q}}}) \longleftrightarrow T_{\ell m}(q) = \frac{1}{\sqrt{4\pi}}\int_{4\pi} d\Omega_{\hat{\mathbf{q}}} T(\mathbf{q})Y^*_{\ell m}(\Omega_{\hat{\mathbf{q}}})$$

$$T_{\ell m}(q_n) \approx \frac{4\pi}{N}\sum_{i=1}^{N}\begin{cases} Y^*_{\ell m}(\Omega_{\hat{\mathbf{q}}_i}) & \text{if } q_i \text{ in bin } n, \\ 0 & \text{otherwise.} \end{cases}$$

similarly for

$M(\mathbf{q})$ $\quad M_{\ell m}(q)$

$$T_{\ell m}(q) = \sqrt{4\pi}\sum_{\ell' m' \ell'' m''} M_{\ell' m'}(q)C_{\ell'' m''}(q)\int_{4\pi} d\Omega_{\hat{\mathbf{q}}} Y^*_{\ell m}(\Omega_{\hat{\mathbf{q}}})Y_{\ell' m'}(\Omega_{\hat{\mathbf{q}}})Y_{\ell'' m''}(\Omega_{\hat{\mathbf{q}}})$$

$$T_{\ell m}(q) = \sum_{\ell'' m''} \tilde{M}_{\ell m \ell'' m''}(q)C_{\ell'' m''}(q) \longrightarrow \mathbf{T}_q = \tilde{\mathbf{M}}_q \cdot \mathbf{C}_q$$ in matrix notation

$$\tilde{M}_{\ell m \ell'' m''} \equiv \sum_{\ell' m'} M_{\ell' m'}(q)(-1)^m \sqrt{(2\ell+1)(2\ell'+1)(2\ell''+1)} \begin{pmatrix} \ell & \ell' & \ell'' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell & \ell' & \ell'' \\ -m & m' & m'' \end{pmatrix}$$

Wigner 3j

$T_{\ell m}(q)$ is a measurement with uncertainty; what's the variance?

$$\Delta^2 T_{\ell m \ell' m'}(q_n) \approx \frac{1}{N(N-1)}\sum_{i=1}^{N}(\sqrt{4\pi}Y^*_{\ell m}(\hat{\mathbf{q}}_i) - T_{\ell m}(q_n))(\sqrt{4\pi}Y^*_{\ell' m'}(\hat{\mathbf{q}}_i) - T_{\ell' m'}(q_n))^*$$

$$\Delta^2 T_{\ell m \ell' m'}(q_n) \approx \frac{1}{N(N-1)}\sum_{i=1}^{N} Y^*_{\ell m}(\hat{\mathbf{q}}_i)Y_{\ell' m'}(\hat{\mathbf{q}}_i) - \frac{1}{N-1}T_{\ell m}(q_n)T^*_{\ell' m'}(q_n).$$

$$\Delta T_{\ell m}(q_n) \approx \sqrt{\Delta^2 T_{\ell m \ell m}(q_n)}.$$

arXiv:0901.3527
Kisiel and Brown

Direct Spherical Harmonic Decomposition II

$$\mathbf{T}_q = \tilde{\mathbf{M}}_q \cdot \mathbf{C}_q$$

arXiv:0901.3527
Kisiel and Brown

Minimize the chi2 $(\mathbf{T}_q - \tilde{\mathbf{M}}_q \cdot \mathbf{C}_q)^T \cdot (\mathbf{\Delta^2 T}_q)^{-1} \cdot (\mathbf{T}_q - \tilde{\mathbf{M}}_q \cdot \mathbf{C}_q)$

$$\boxed{\mathbf{C}_q = \mathbf{\Delta^2 C}_q \cdot \tilde{\mathbf{M}}_q^T \cdot (\mathbf{\Delta^2 T}_q)^{-1} \cdot \mathbf{T}_q \qquad \mathbf{\Delta^2 C}_q = (\tilde{\mathbf{M}}_q^T \cdot (\mathbf{\Delta^2 T}_q)^{-1} \cdot \tilde{\mathbf{M}}_q)^{-1}}$$

For **one** q bin

This assumes the variance on M_q is ~0 for large mixing depth.

Assumes elements complex numbers based on the $\begin{pmatrix} R & -I \\ I & R \end{pmatrix}$ format.

$\tilde{\mathbf{M}}_q^T \rightarrow \tilde{\mathbf{M}}_q^*$ if one uses complex numbers directly as matrix elements.

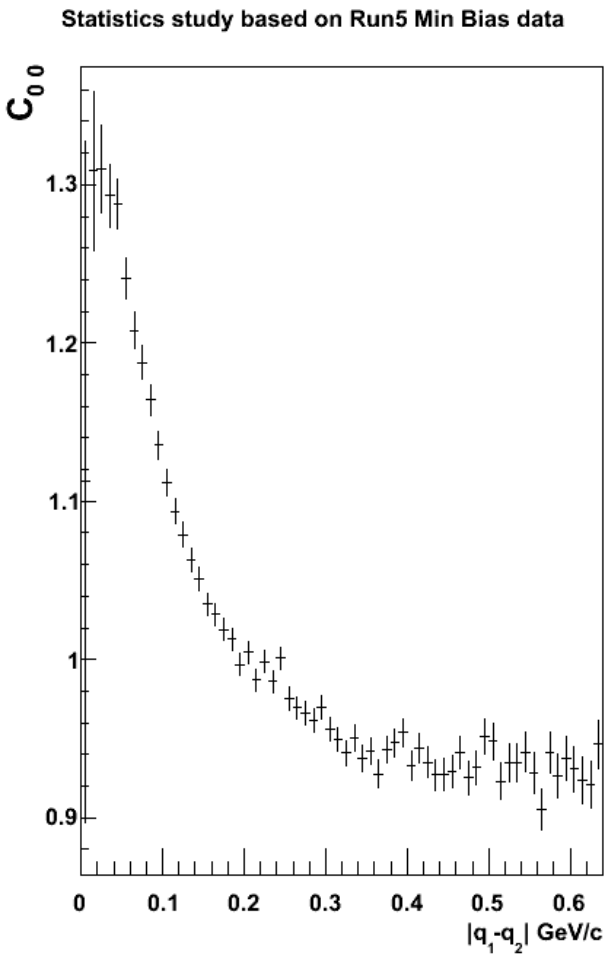
There can be experimental corrections such as:

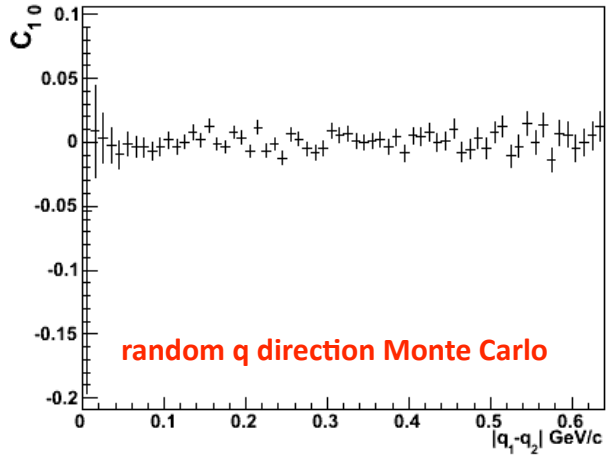
- Merging/splitting Monte Carlo corrections
  - applied directly as weights to Y_{lm}
- Purity correction, *P*
  - decompose *P* spherical harmonics and divide $C_{measured}$

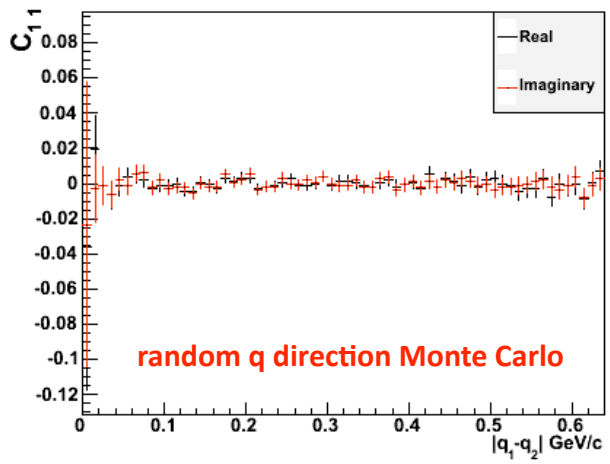
I personally see no reason to do an analysis using the traditional Cartesian binning to Y_{lm} . If you do, please let me (and the rest of the world) know.

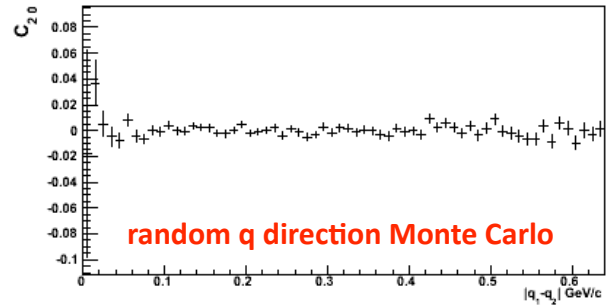
Statistic Study: Glimpse at the Future

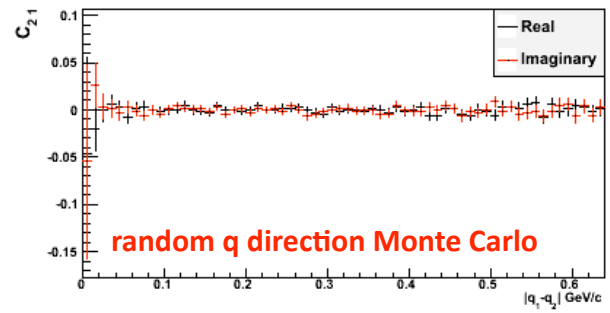
Use foreground and background q count distributions from real data.
Assume random q direction.

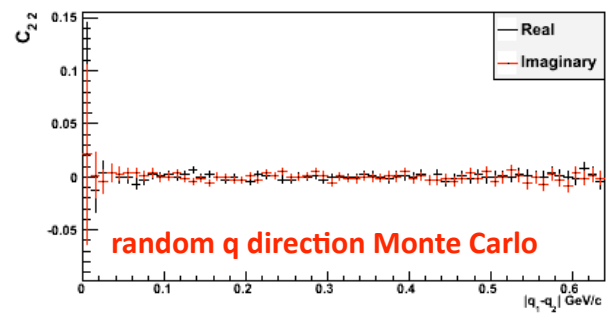












Errors are not correlated q bin to bin, but are C_{lm} to $C_{l'm'}$

Summary

- Pion correlations in minimum biased collisions for PHENIX west arm acceptance do not appear to show significant energy momentum correlations within statistics.
- Measured radii consistent with centrality extrapolation from Au+Au
- Plenty still to look at (spherical harmonic decomposition, multiplicity dependence…)
- Jet selected events on the horizon (will need tools to deal with energy-momentum contamination)

Jet Selected p+p Teaser

- An early look at events with a reconstructed jet show this should be interesting.

Aside on likelihood fitting

$$\chi^2 = -2\ln(\lambda) \tag{1}$$

where the likelihood ratio $\lambda = P(N|\mu)/P(\mu|\mu)$

or in Bayesian statistics $\lambda = P(\mu|N)/P(\mu|\mu)$

where P is the probability μ is the expectation value and N is the measurement. For Poisson statistics $P(\mu|N) = \frac{\mu^N}{N!}e^{-\mu}$, and we can calculate the joint probability for a correlation function $C = \mu/\nu$ where μ is the mean of the actual pair measurement A and ν is the mean of the mixed event Background measurement B.

$$P(C|A,B) = \int\int \frac{\mu^A}{A!}e^{-\mu}\frac{\nu^B}{B!}e^{-\nu}(C-\frac{\mu}{\nu})d\mu d\nu$$
$$= \frac{(A+B+1)!}{A!B!}\frac{C^A}{(C+1)^{A+B+2}}$$

Not in
Annu. Rev. Nucl. Part. Sci.
2005. 55:357–402

and

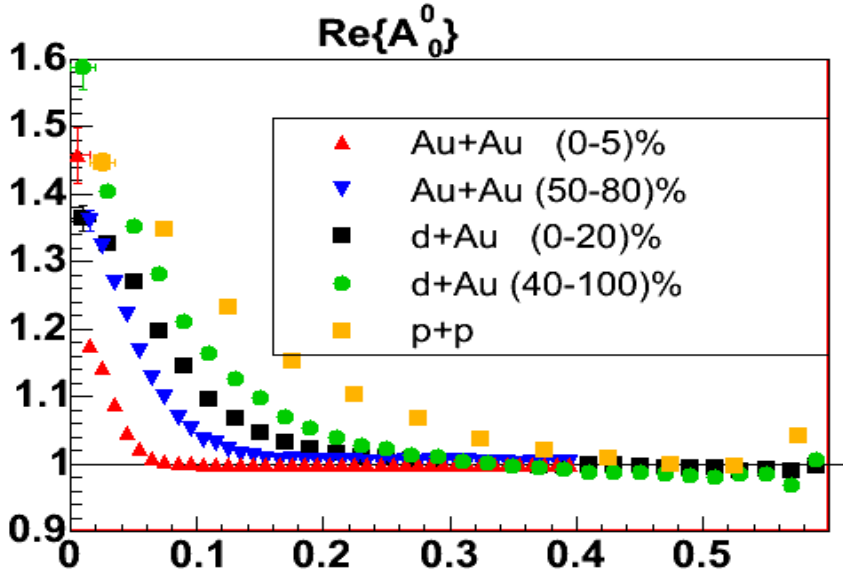
$$\lambda = \frac{P(C|A,B)}{P(A/B|A,B)} = \frac{C^A}{(A/B)^A}\frac{(A/B+1)^{A+B+2}}{(C+1)^{A+B+2}} \tag{2}$$

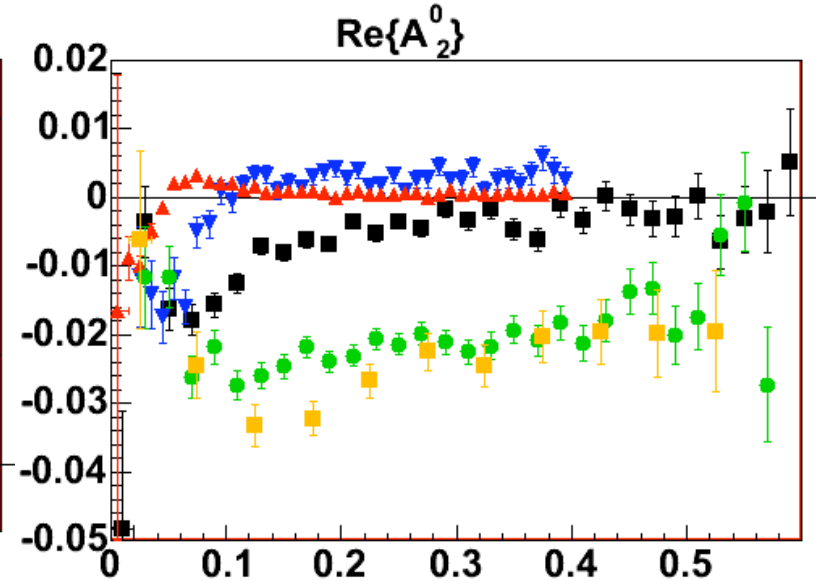
therefor

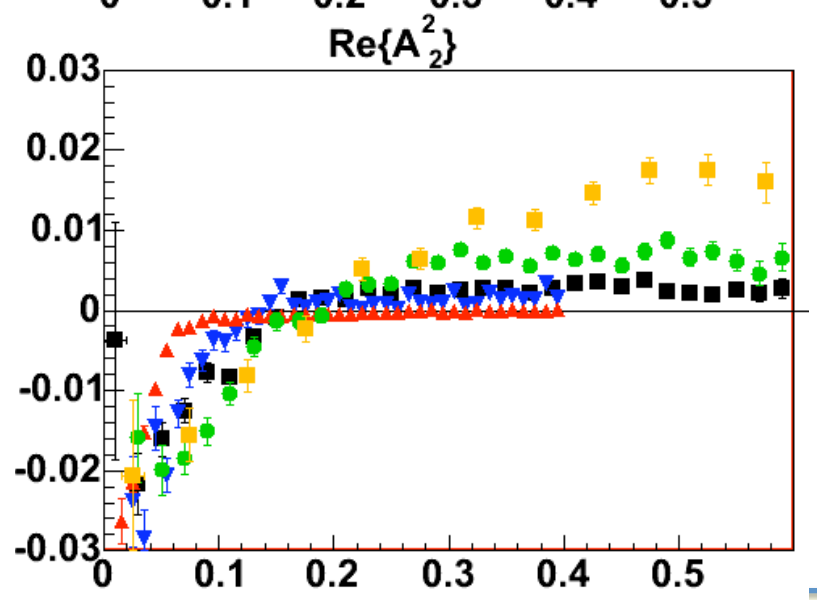
$$-\frac{1}{2}\chi^2 = \ln(\lambda) = A\ln\left(\frac{C(A+B)}{A(C+1)}\right) + (B+2)\ln\left(\frac{A+B}{B(C+1)}\right) \tag{3}$$

For $A = 0$, the first term of Eq. 3 is 0 since $n\ln(n) \to 0$ as $n \to 0$.

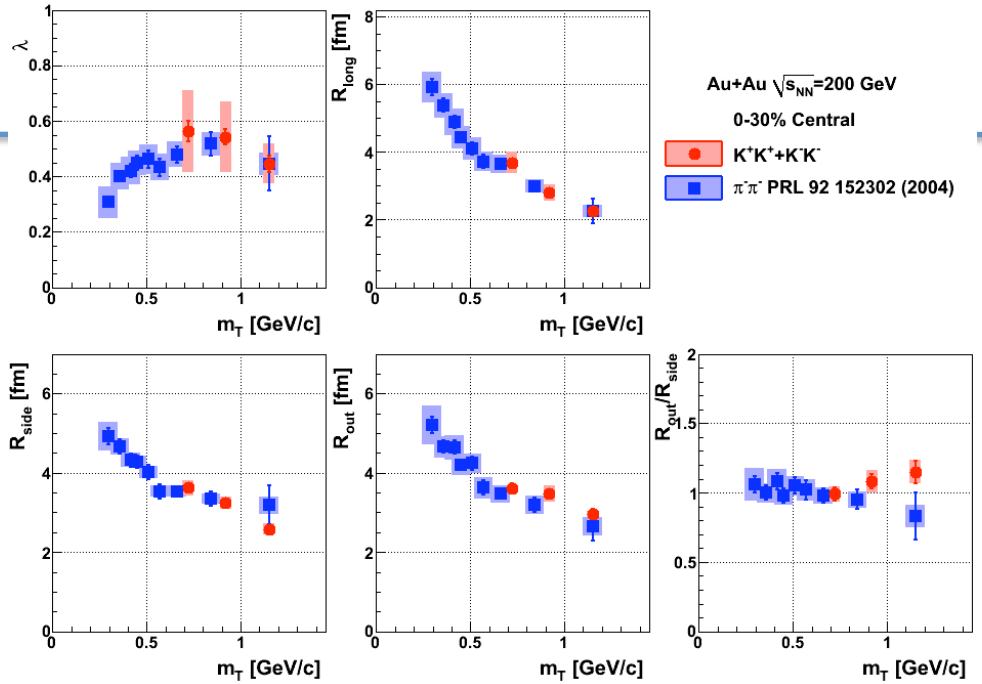
Baseline in p+p, d+Au and Au+Au

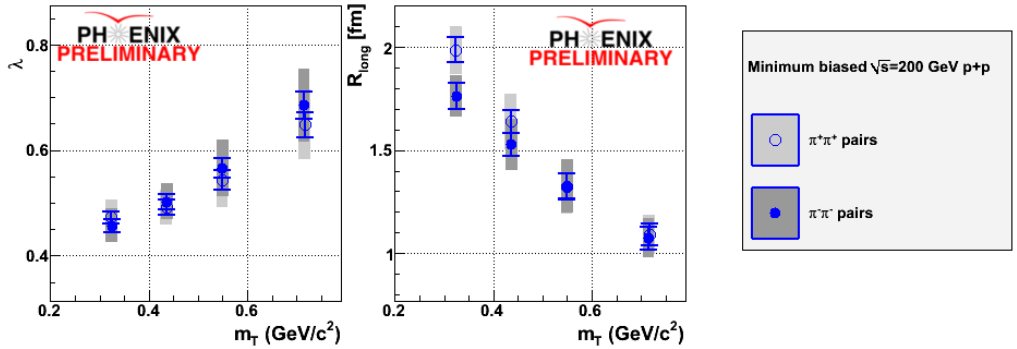


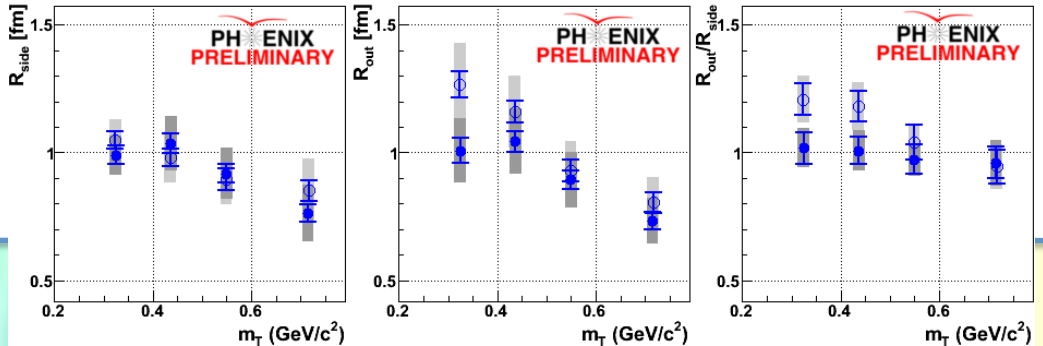


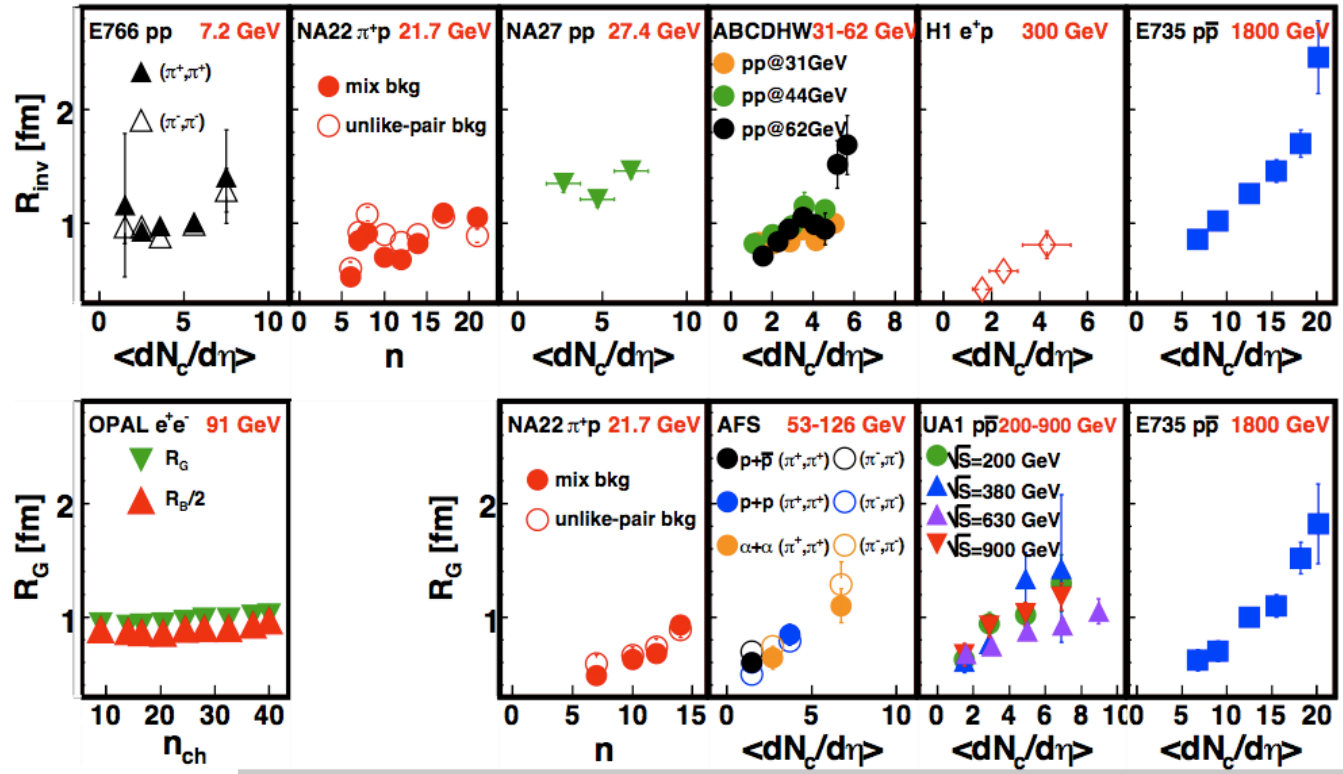


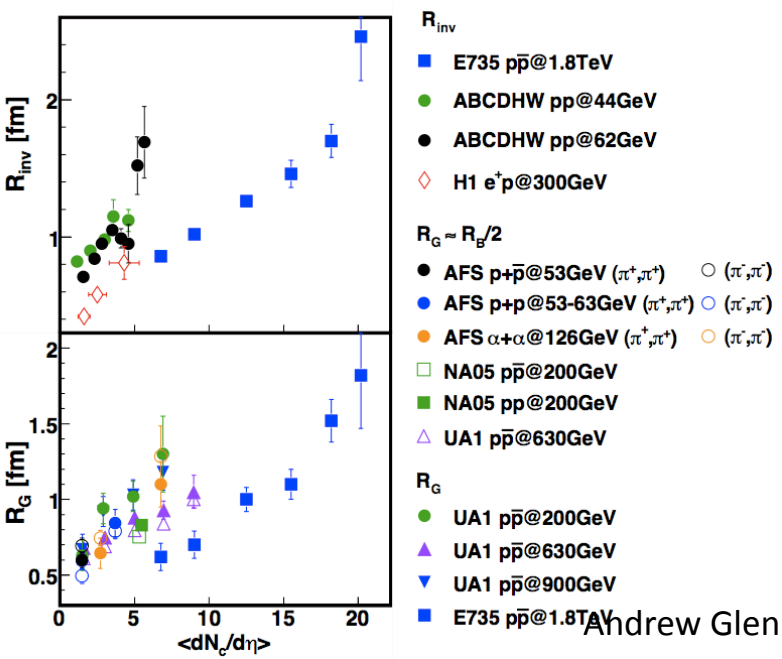
- Some coefficients don't vanish for large Q
- Bias from zero is larger for more peripheral collisions
- This problem is getting more significant for small systems (pp & dAu)
- It can be due to
  - an experimental artifact
  - long range correlations (e.g. Jets)
  - problem with momentum conservation ?
  - or ?


Au+Au $\sqrt{s_{NN}}$=200 GeV
0-30% Central
$K^+K^+ + K^-K^-$
$\pi^-\pi^-$ PRL 92 152302 (2004)
λ
R_{long} [fm]
R_{side} [fm]
R_{out} [fm]
R_{out}/R_{side}
m_T [GeV/c]


PHENIX
PRELIMINARY
Minimum biased $\sqrt{s}$=200 GeV p+p
$\pi^+\pi^+$ pairs
$\pi^-\pi^-$ pairs
λ
R_{long} [fm]
m_T (GeV/c^2)


PHENIX
PRELIMINARY
R_{side} [fm]
R_{out} [fm]
R_{out}/R_{side}
m_T (GeV/c^2)





arXiv:0901.4078