# Measurement of event-by-event fluctuations and order parameters in PHENIX

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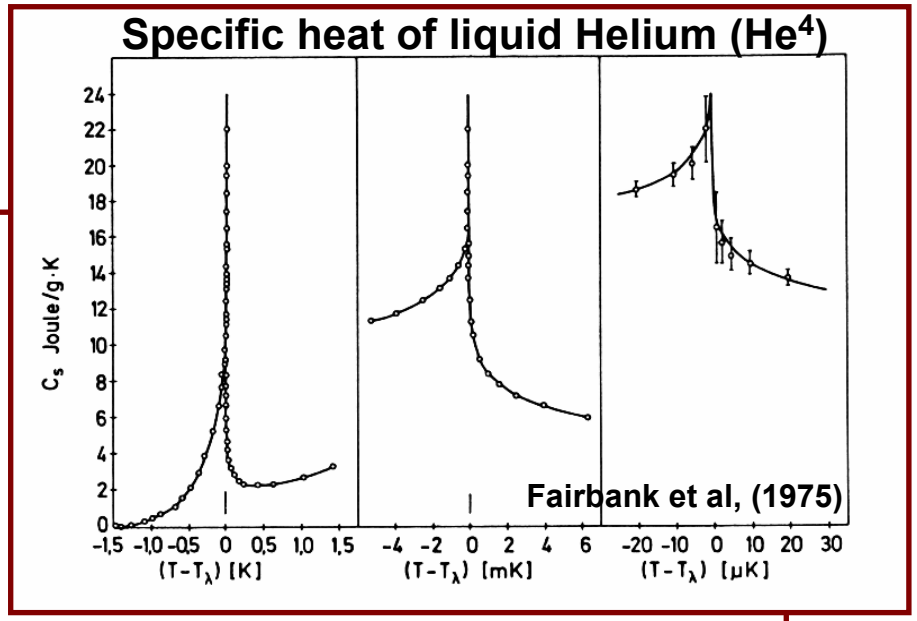
# Phase transitions



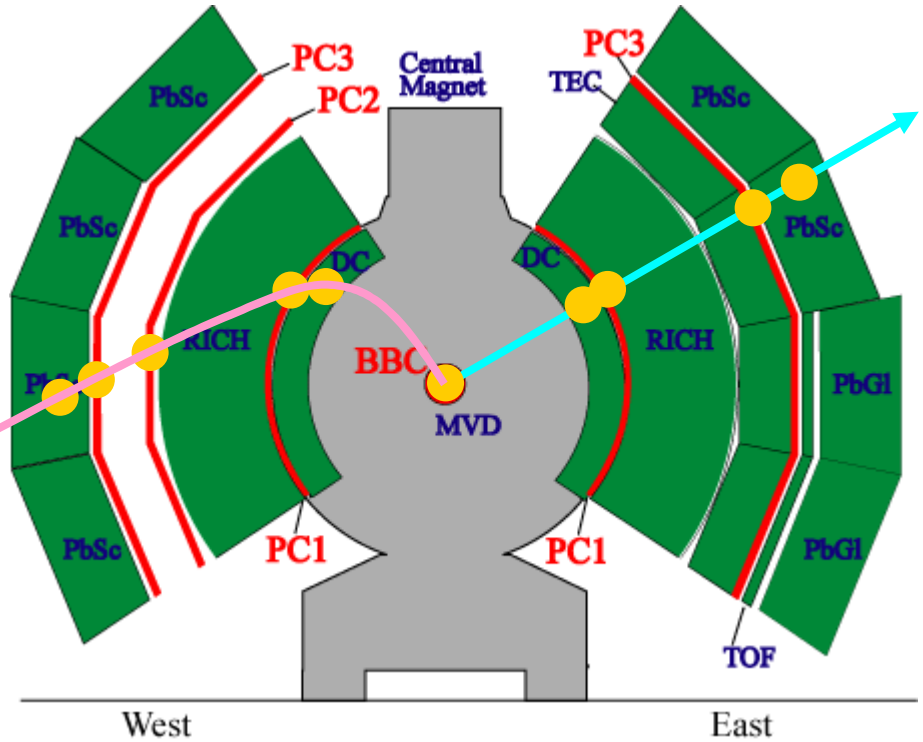

- According to the classical classification of the phase transition, the order of phase transition is defined by discontinuities in derivatives of free energy.

- In this aspect of bulk property, discontinuity in thermodynamic variables or order parameters as a function of the temperature or time evolution are available to search for the critical point of phase.

- In particular, the second order phase transitions are often accompanied by the divergence with respect to thermodynamic variables as a results of critical phenomena.

# Event-by-event fluctuations in heavy-ion collisions measured by PHENIX

- Some thermodynamic variables as order parameters of phase can be obtained from event-by-event fluctuations.
- Particles correlation length
  - scale dependence of multiplicity fluctuations
- Specific heat
  - temperature fluctuations from average $p_T$ fluctuations.
    PRL. 93 (2004) 092301
    **M. J. Tannenbaum : poster #120**
- We have performed measurements of several fluctuations to explore the QCD phase transition using the PHENIX detector at RHIC.




Geometrical acceptance

$\Delta\eta < 0.7$

$\Delta\phi < \pi$

Momentum range

$0.2 < p_T < 2.0\, GeV/c$

# Multiplicity fluctuations by variance

Average of multiplicity distribution

$$< n >= \sum n \cdot P(n)$$

Variance of multiplicity distribution

$$Var(n) = \sum (n - < n >)^2 P(n) = < n^2 > - < n >^2$$

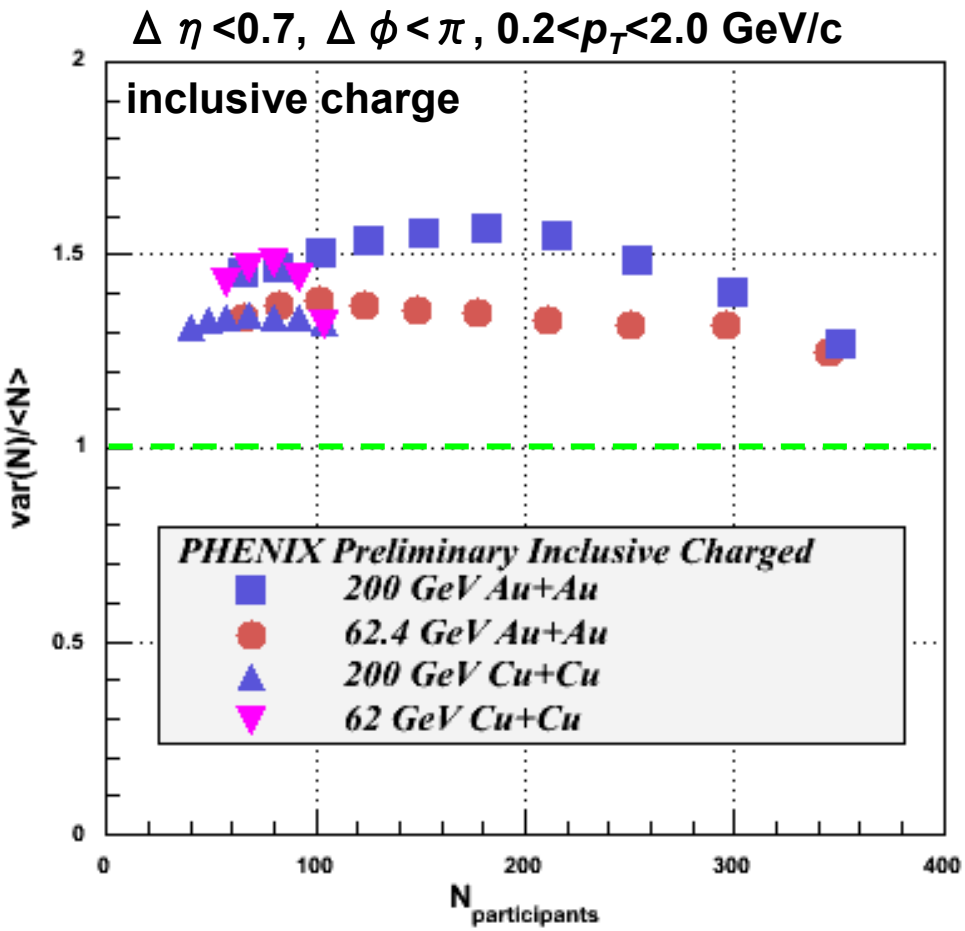
Normalized variance of multiplicity distribution

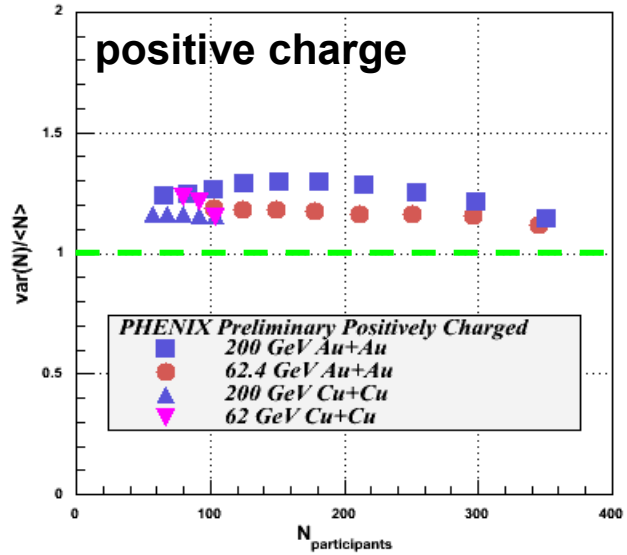
$$Var(n)/n = \frac{< n^2 > - < n >^2}{n}$$

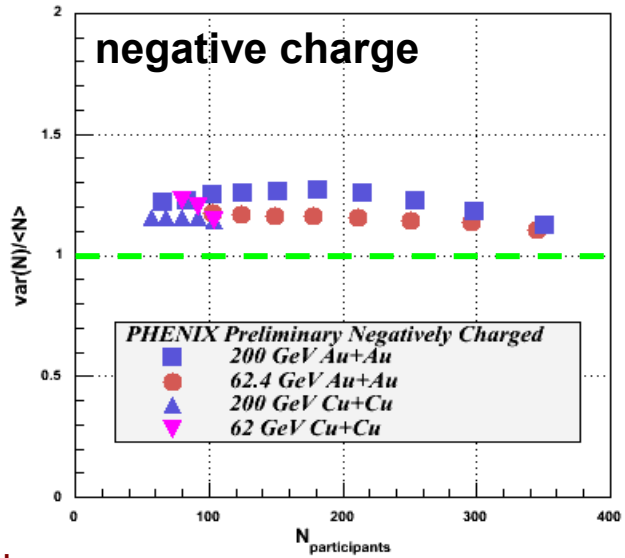
Normalized variance is used as an observable of multiplicity fluctuation. In the case of Poisonnian distribution, the variance equals mean value, then normalized variance indicates 1.

# Normalized variance vs. participants

J. T. Mitchell : poster #110



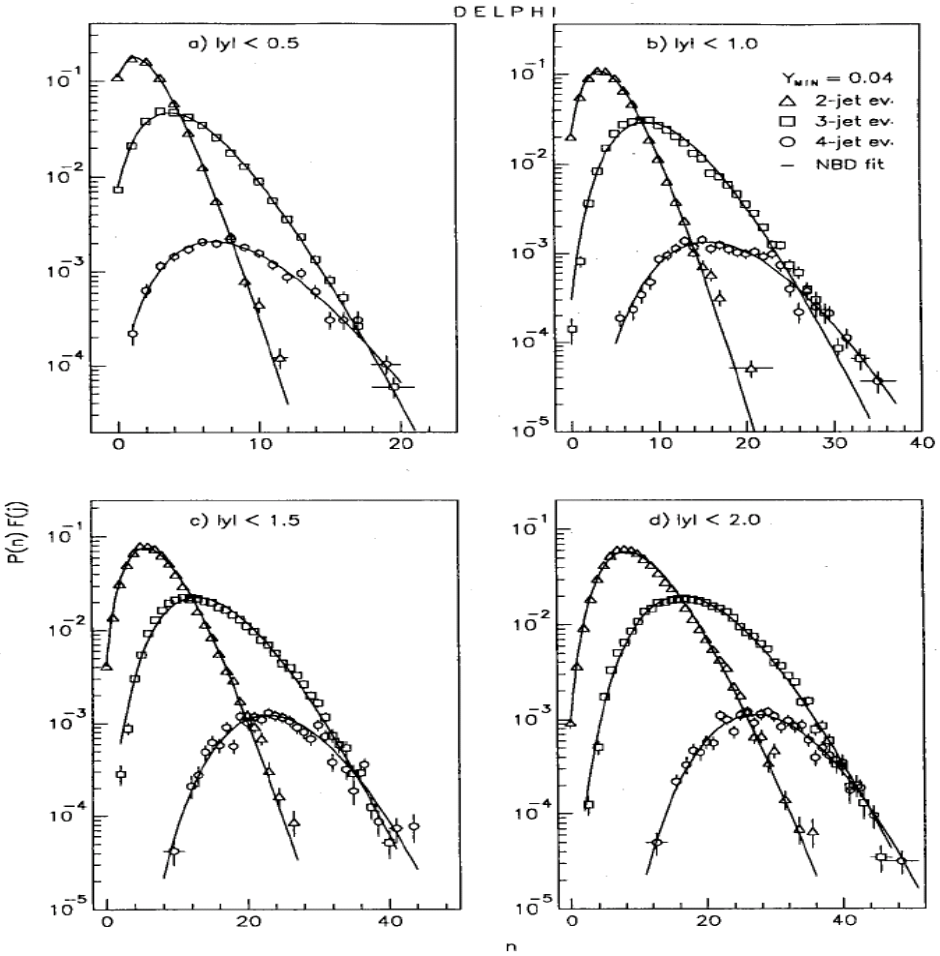



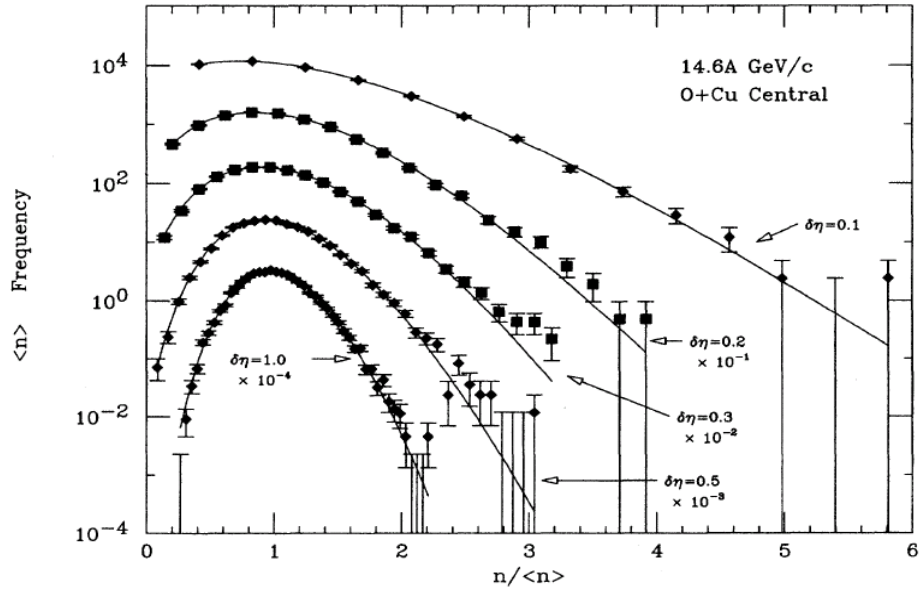

Deviation from the Poisonnian.
No charge dependences.
Similar behaviors of Cu+Cu 62GeV to SPS.

# Charged particle multiplicity distributions and negative binomial distribution (NBD)

**DELPHI: $Z^0$ hadronic Decay at LEP**
**2,3,4-jets events**




**E802: $^{16}$O+Cu 16.4AGeV/c at AGS**
**most central events**




**[DELPHI collaboration] Z. Phys. C56 (1992) 63**
**[E802 collaboration] Phys. Rev. C52 (1995) 2663**

Universally, hadron multiplicity distributions conform to NBD in high energy collisions.

# Negative binomial distribution (NBD)

$$P_n = \mu^{-n} / (1+\mu)^{n+1}$$

Bose-Einstein distribution
$\mu$ : average multiplicity

$$P_n^{(k)} = \frac{\Gamma(n+k)}{\Gamma(n-1)\Gamma(k)} \left( \frac{\mu / k}{1+\mu / k} \right)^n \frac{1}{(1+\mu / k)^k}$$

NBD

$$\frac{\sigma^2}{\mu^2} = \frac{1}{\mu} + \frac{1}{k}$$

$$\sigma \equiv \sqrt{<n^2> - <n>^2}$$

$$\frac{1}{k(\delta)} = \frac{\sigma^2}{\mu^2} - \frac{1}{\mu} = F_2(\delta) - 1$$

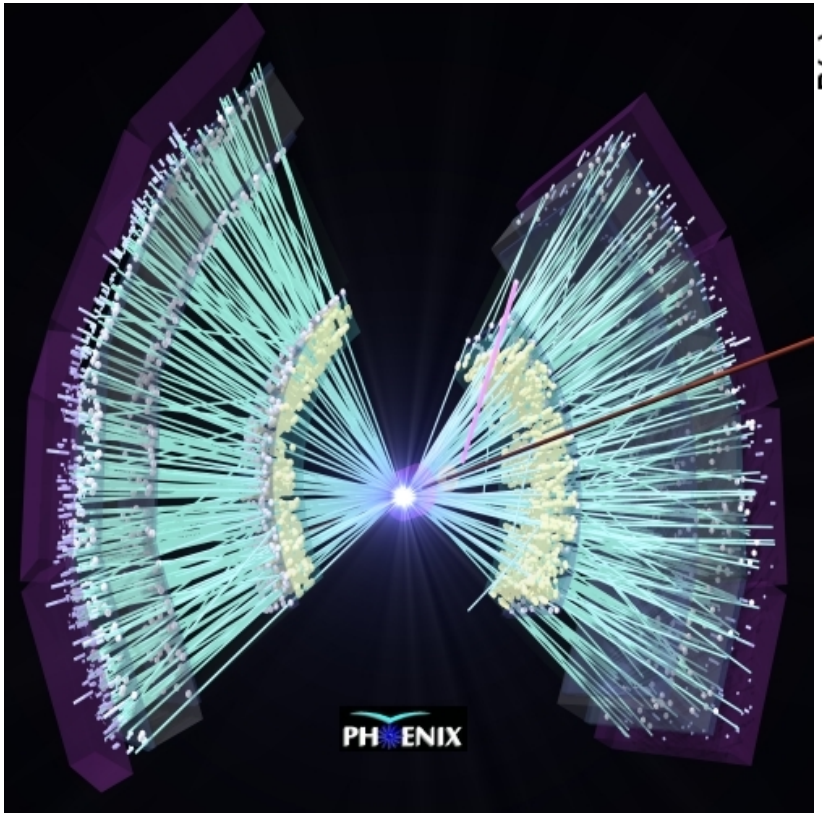
$$F_2(\delta) = \frac{<n>^2 - <n>}{<n>^2}$$

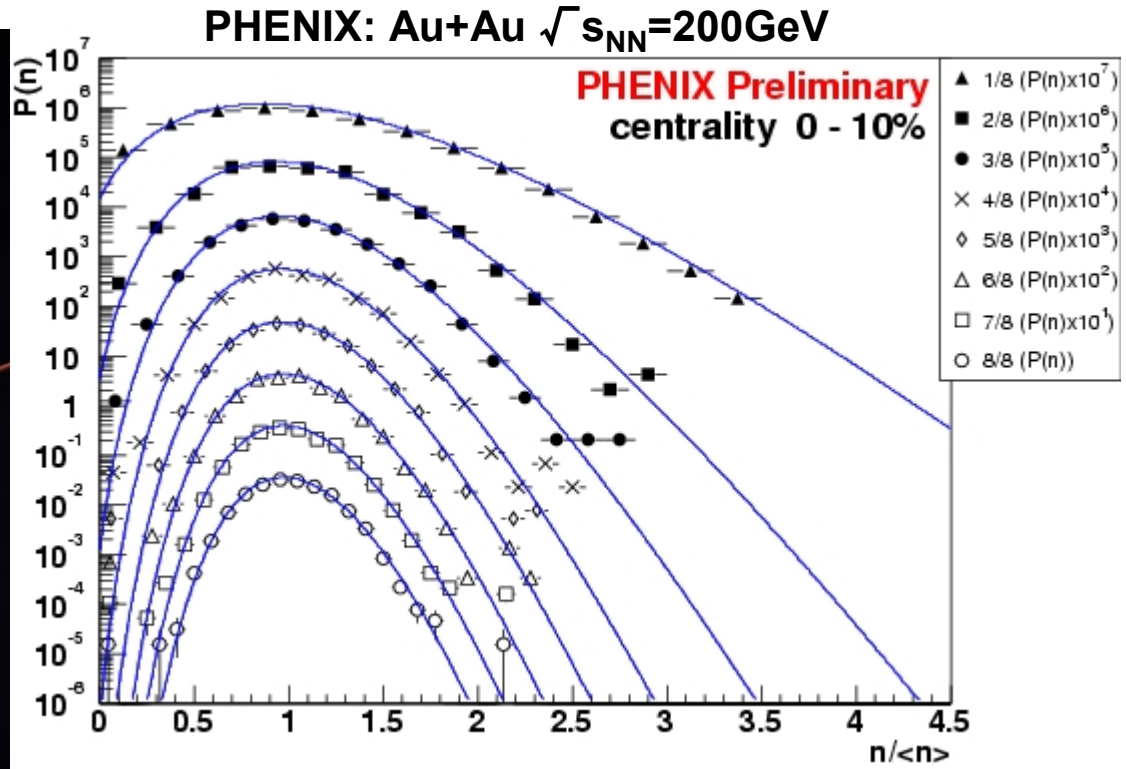
$F_2$ : second order normalized factorial moment

NBD correspond to multiple Bose-Einstein distribution and the parameter k indicates the multiplicity of Bose-Einstein emission sources.

NBD also corresponds to the Poison distribution with the infinite k value in the statistical mathematics.

# Charged particle multiplicity distributions in PHENIX



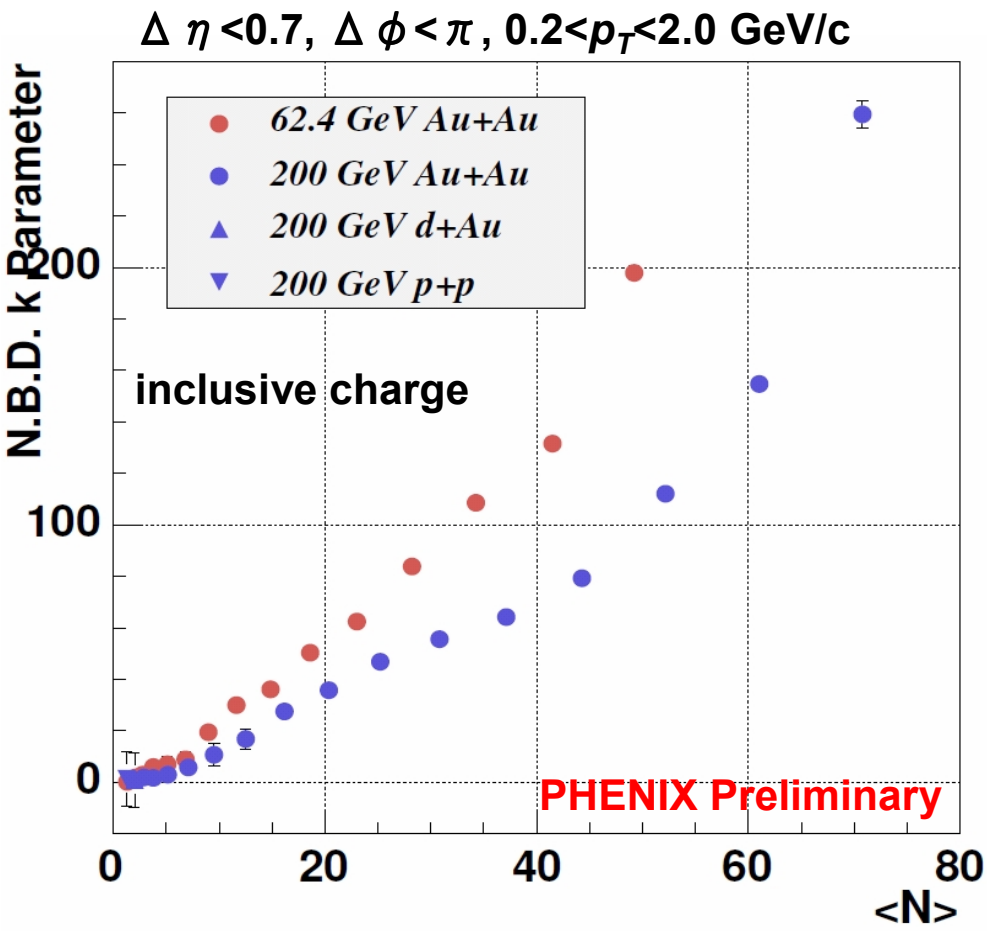



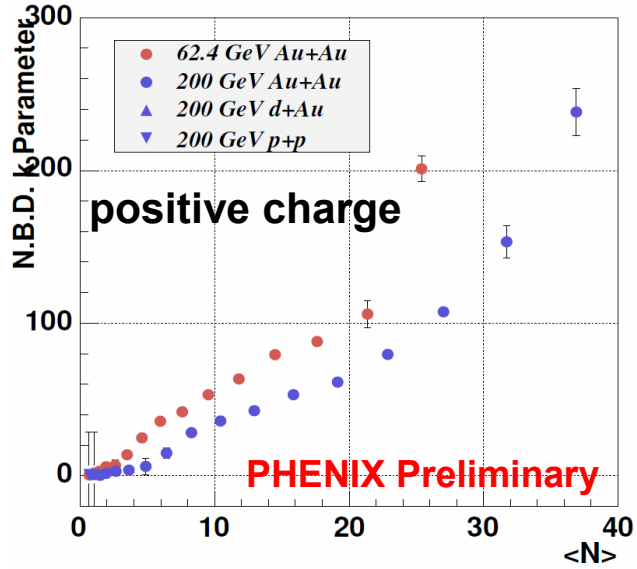

Multiplicity distributions observed in Au+Au, Cu+Cu, d+Au and p+p collisions at PHENIX also conform to the negative binomial distribution.

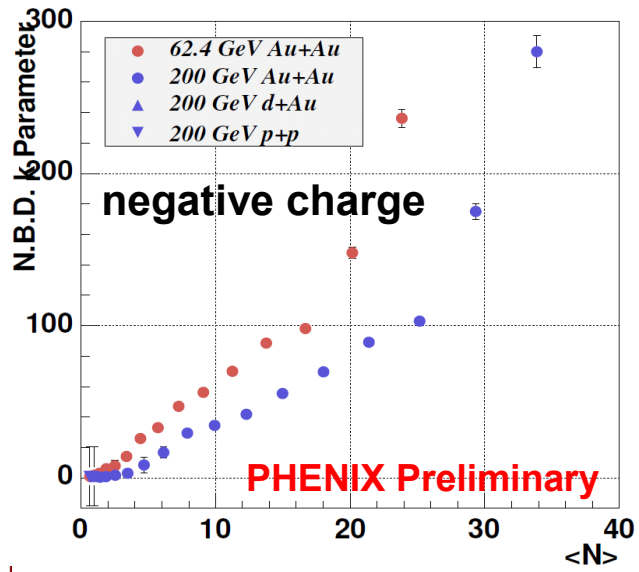
**No magnetic field**
**$\Delta\eta$ <0.7, $\Delta\phi < \pi/2$**

$\delta\eta$ = 0.09 (1/8) : P(n) x $10^7$
$\delta\eta$ = 0.18 (2/8) : P(n) x $10^6$
$\delta\eta$ = 0.35 (3/8) : P(n) x $10^5$
$\delta\eta$ = 0.26 (4/8) : P(n) x $10^4$
$\delta\eta$ = 0.44 (5/8) : P(n) x $10^3$
$\delta\eta$ = 0.53 (6/8) : P(n) x $10^2$
$\delta\eta$ = 0.61 (7/8) : P(n) x $10^1$
$\delta\eta$ = 0.70 (8/8) : P(n)

# NBD k parameters as a function of average multiplicity

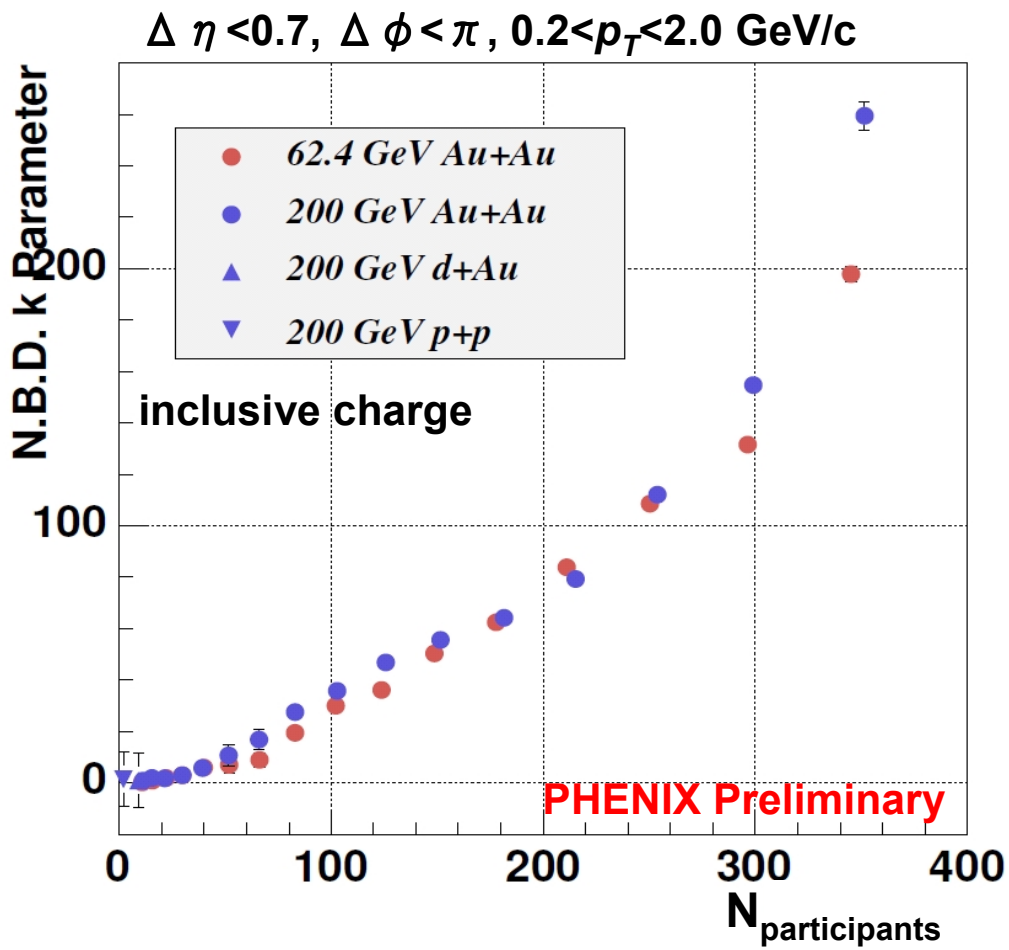


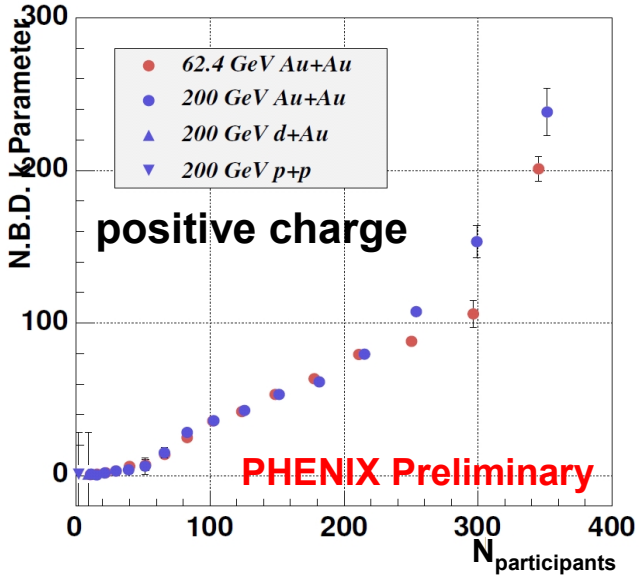



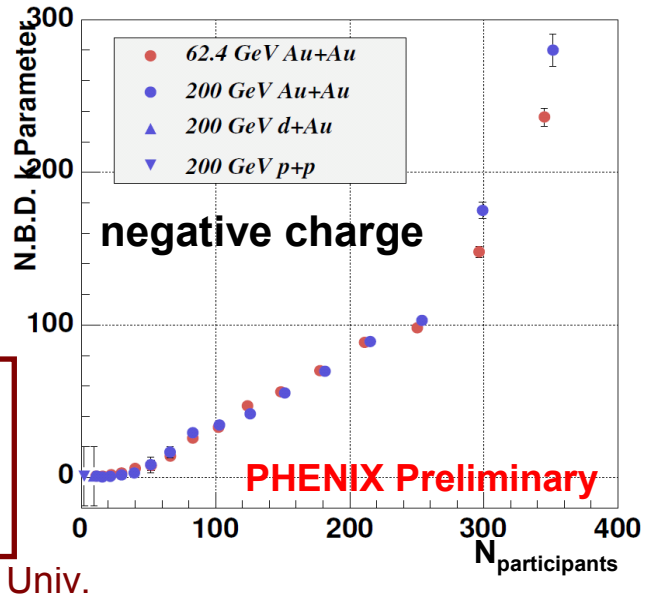




There are the differences about the average multiplicity dependence of NBD k parameters between the 200GeV and 62.4 GeV.

# NBD k parameters as a function of number of participants



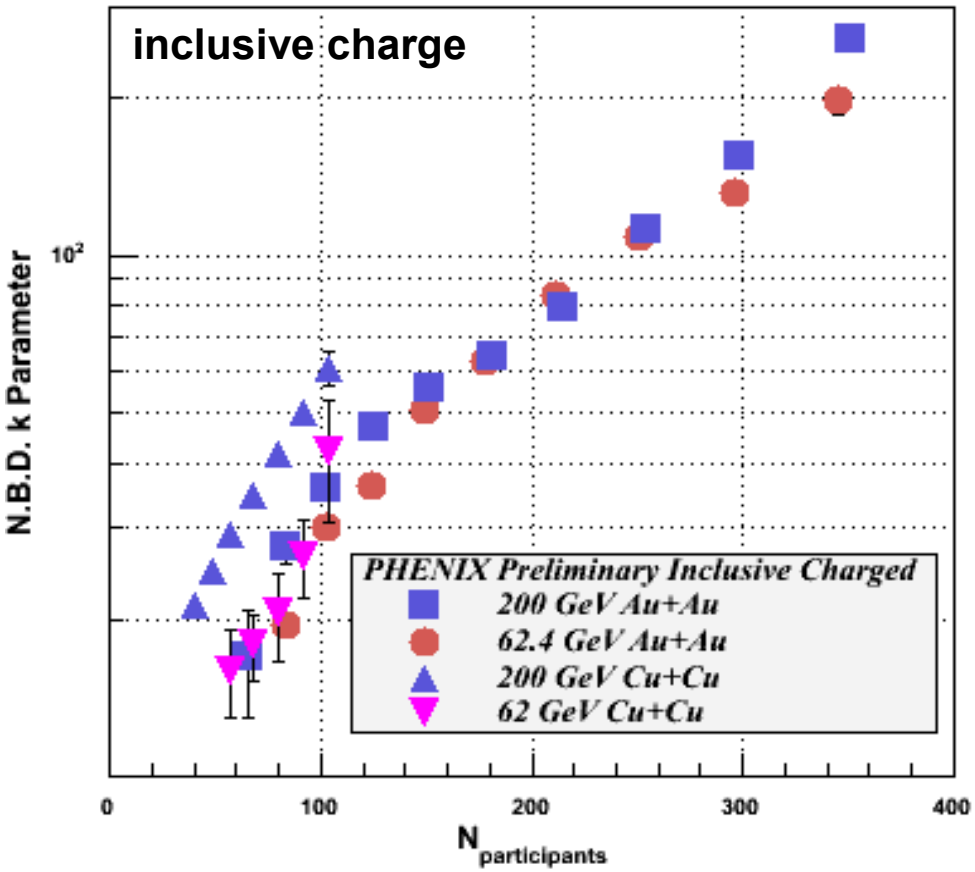



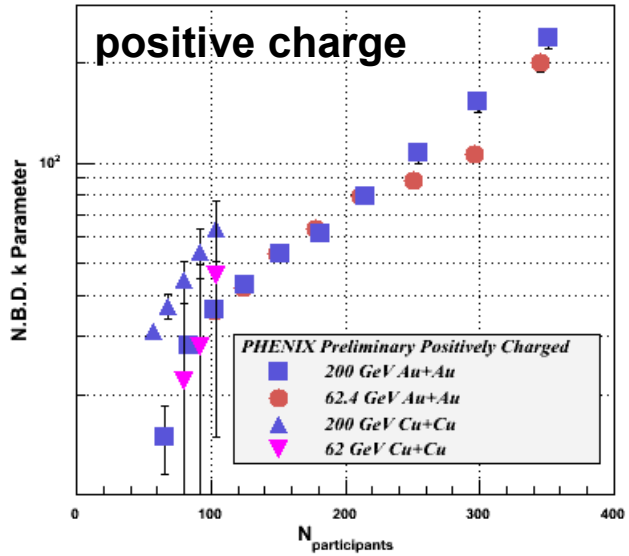



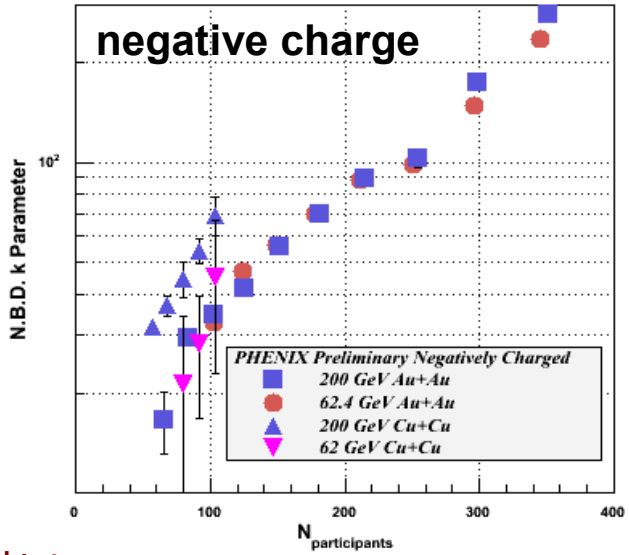

NBD k parameters as an observable of multiplicity fluctuation are not scaled by the average multiplicity but scaled by the number of participants.

# NBD k parameters in Cu+Cu

$\Delta\eta$ <0.7, $\Delta\phi < \pi$, 0.2<$p_T$<2.0 GeV/c

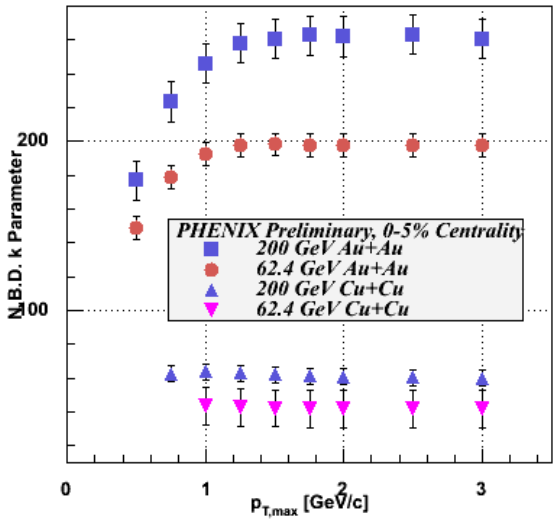


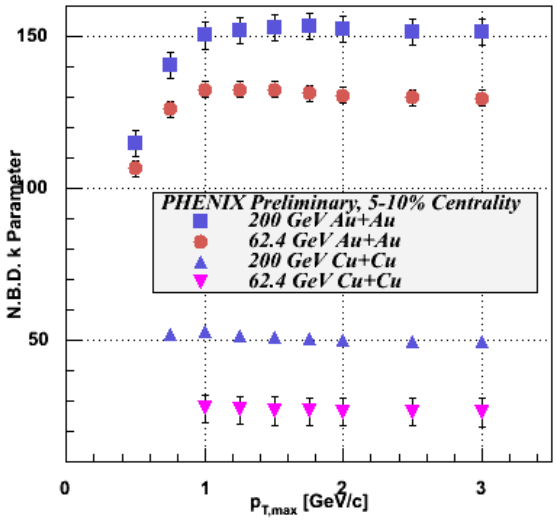



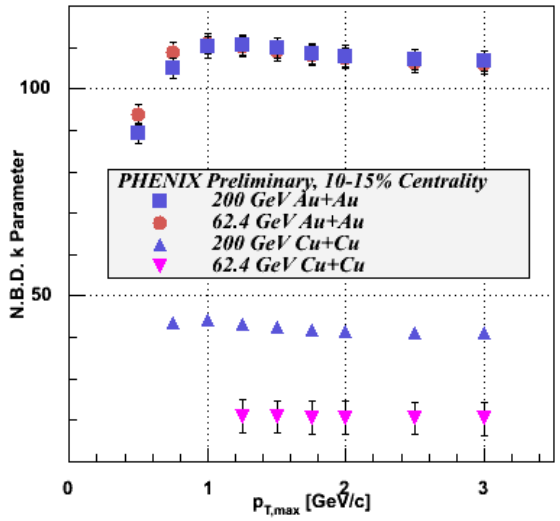



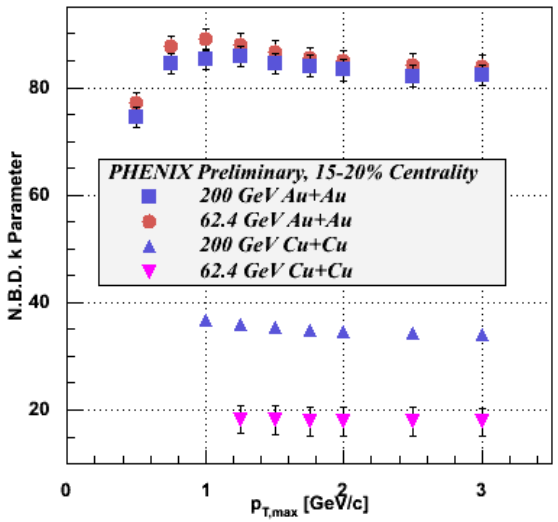

NBD k parameters are scaled by system size in Au+Au, but not scaled in Cu+Cu.

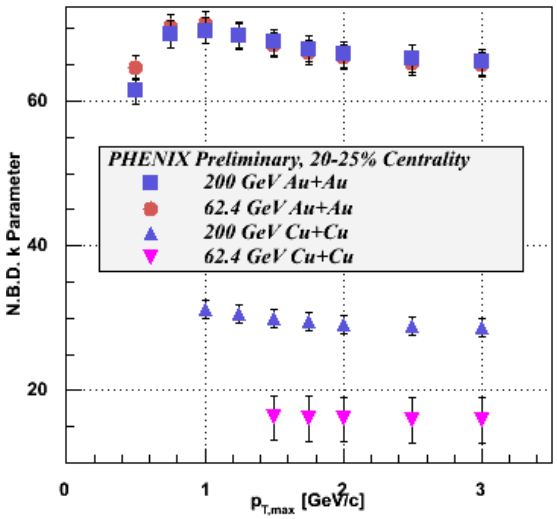
# NBD k parameters as functions of $\delta p_T$ ($p_T$>0.2 GeV/c)

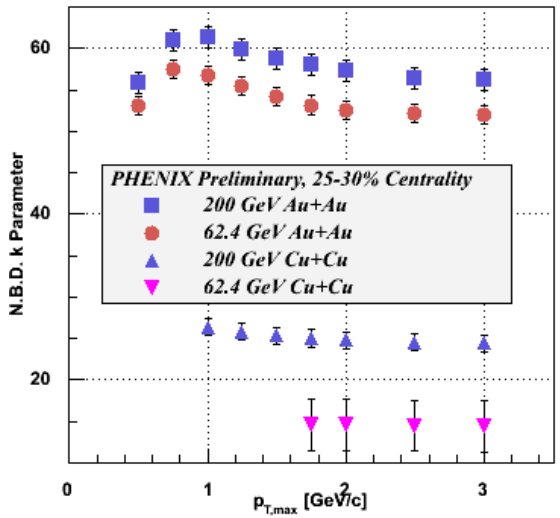


# Extraction of two particle correlation

Normalized correlation function

$$R_2(y_1, y_2) = \frac{C_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} = \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1$$

$\rho_1(y)$ : inclusive single particle density

$\rho_2(y_1, y_2)$ : inclusive two-particle density

$C_2(y_1, y_2)$ : two-particle correlation function

Relation with NBD k

$$\frac{1}{k(\delta\eta)} = F_2 - 1 = K_2 = \frac{\int^{\delta\eta} C_2(y_1, y_2)\, dy_1 dy_2}{\int^{\delta\eta} \rho_1(y_1)\rho_1(y_2)\, dy_1 dy_2}$$

**Used in E802 : PRC, 44 (1991) 1629**

$$R_2 = R_0 e^{-|y_1 - y_2|/\xi} \quad : \quad \frac{1}{k(\delta\eta)} = F_2 - 1 = \frac{2R_0\xi^2[\delta\eta/\xi - 1 + e^{-\delta\eta/\xi}]}{\delta\eta^2}$$

**Two component model**

$$R_2 = e^{-|y_1 - y_2|/\xi} + b \quad : \quad \frac{1}{k(\delta\eta)} = F_2 - 1 = \frac{2\xi^2[\delta\eta/\xi - 1 + e^{-\delta\eta/\xi}]}{\delta\eta^2} + \frac{b}{2}$$

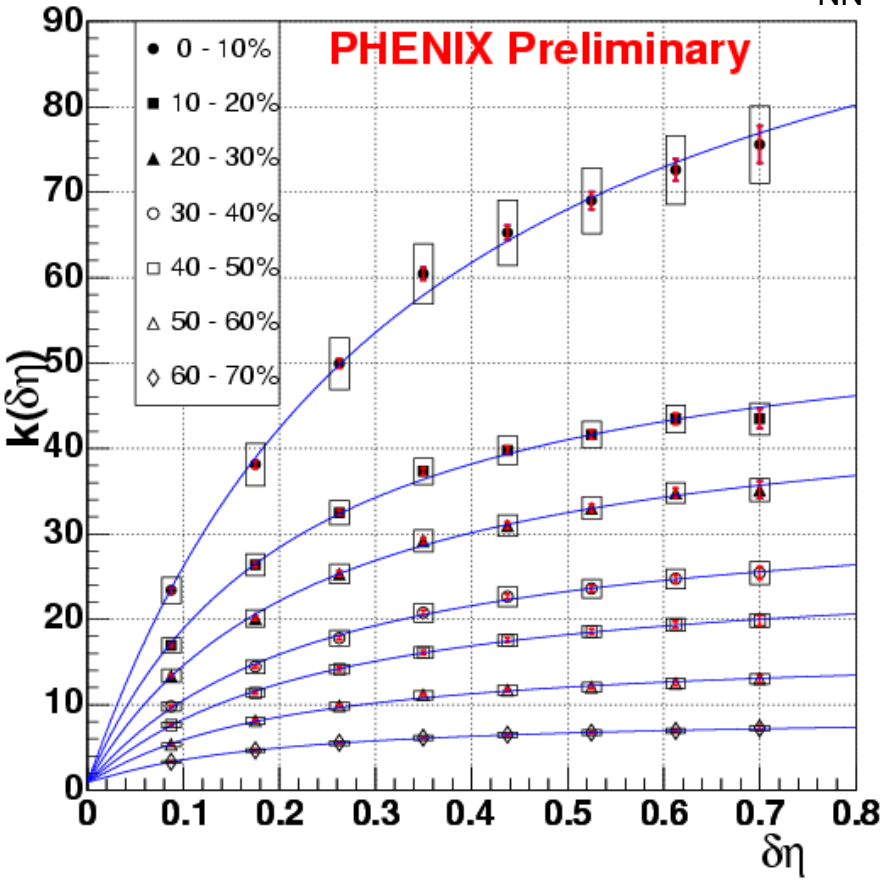
$\xi$ : Two particle correlation length

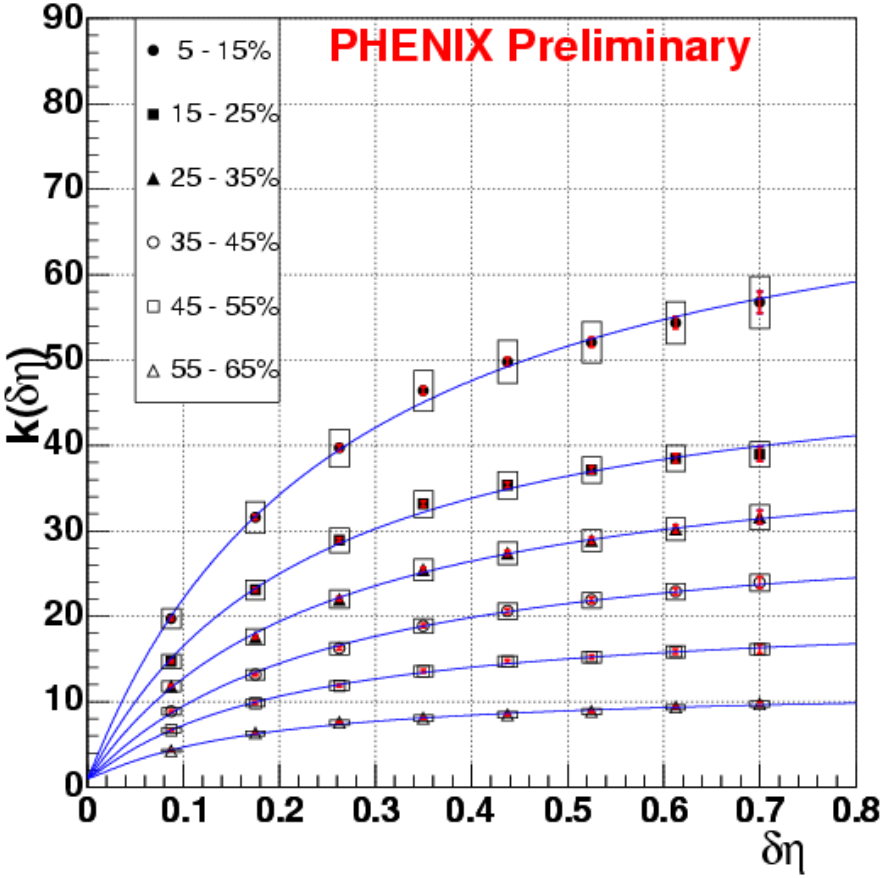
$b$ : Strength of long range correlation

# NBD k and pseudo rapidity gap

- Two component model well agree with data.
- Correlation function dose not go to 0 at $\delta\eta$ equal 0. It dose not suggest the intermittency effect.

PHENIX: Au+Au $\sqrt{s_{NN}}$=200GeV, $\Delta\eta$<0.7, $\Delta\phi<\pi/2$



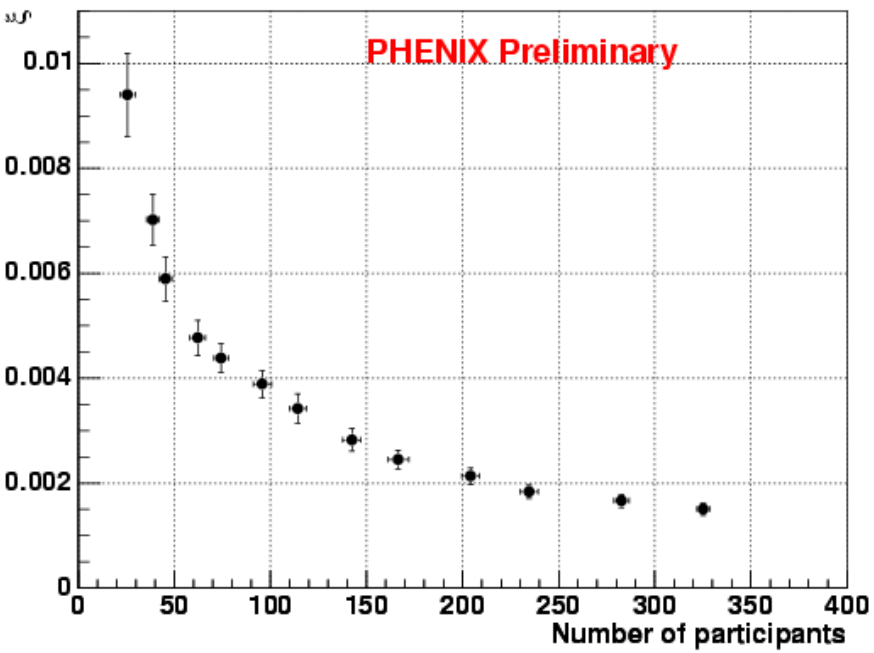



# Participants dependence of $\xi$ and $b$

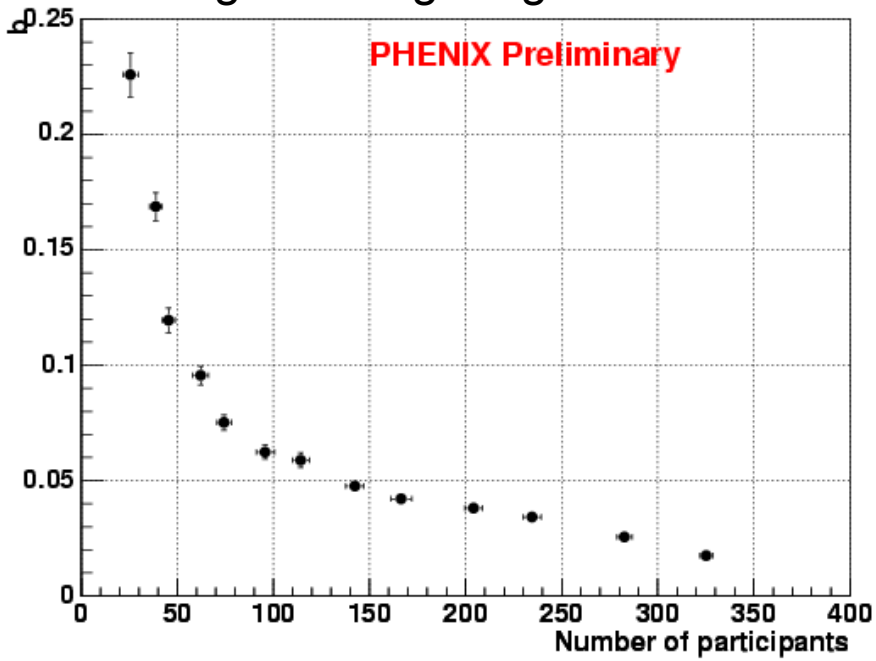
- Smaller value of two particle correlation length have been observed at RHIC energy as compared to the past experiments.
  - Low density p+p collisions (UA5, p+p $\sqrt{s}$ = 540 GeV : $\xi$ = 2.9)
  - Low energy N+N collisions (E802 O+Cu 14.6AGev/c : $\xi$ = 0.18±0.05)
- Both $\xi$ and b decrease with increasing the number of participants.

PHENIX: Au+Au $\sqrt{s_{NN}}$=200GeV

Two particle correlation length



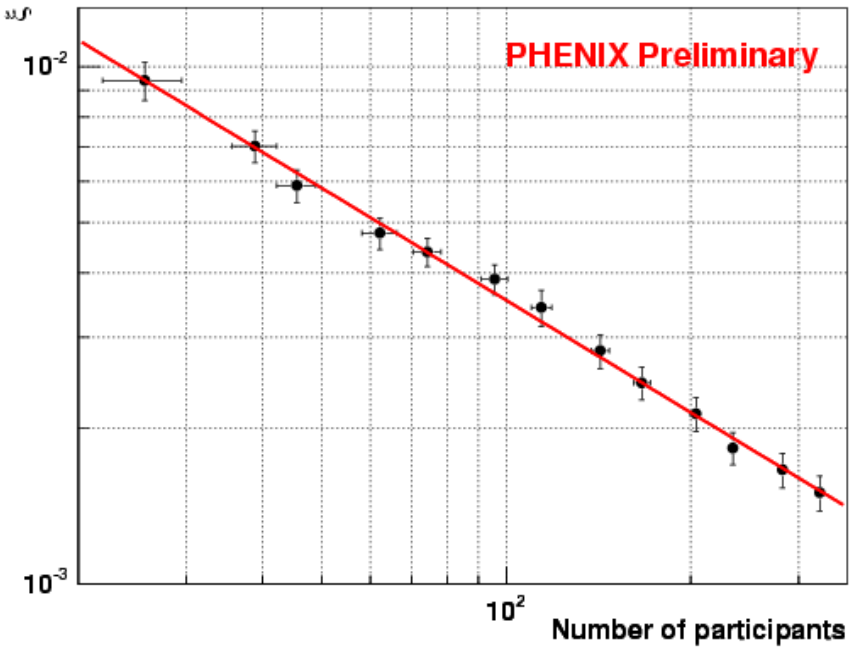

Strength of long range correlations



# Linearity in the log-log plot
# $\xi$ vs. number of participants

PHENIX: Au+Au $\sqrt{s_{NN}}$=200GeV

Two particle correlation length




$$T \propto N_{part}$$

$$\xi \propto |T - T_c|^{\alpha} \propto |N_{part}|^{\alpha}$$

$$\log(\xi) \propto \alpha \log(N_{part})$$

$\alpha$ = -0.72 ± 0.032

Linear behavior of the correlation length as a function of the number of participants have been observed in the logarithmic scale. It suggests hadron two particle correlation length have a information of the thermodynamical systems by assuming the proportionality between the number of participants and temperature.

# Conclusions

- A systematic study on charged particle multiplicity fluctuations have been performed in Au+Au, Cu+Cu, d+Au and p+p collisions with respect to two collision energy of 200GeV and 62GeV.
- Multiplicity distributions measured by PHENIX also agree with the negative binomial distributions at the RHIC energy.
- Multiplicity fluctuations by the NBD k parameters are not scaled by the average multiplicity but scaled by the number of participants or system size in Au+Au collisions.
- Scale dependence of NBD k parameters are presented with respect to pseudo rapidity gap and transverse momentum range.
- Two particle correlation length have been observed by the two component model from the multiplicity fluctuations.
- Extracted correlation length have a linearity as a function of the number of participants in the logarithmic scale (log-log plot).

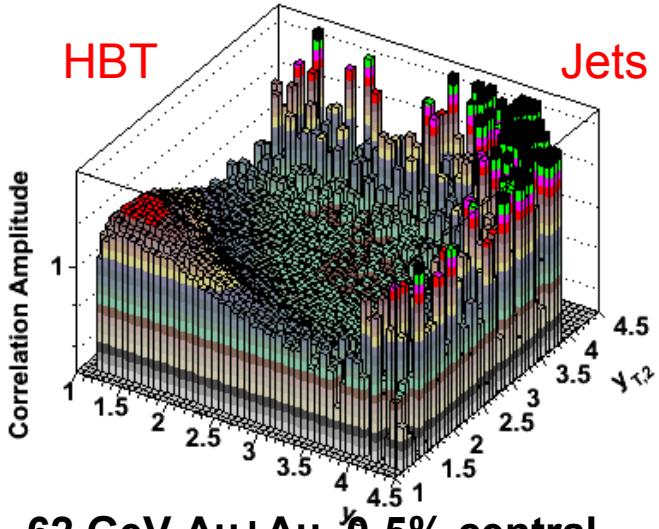
# PHENIX posters on fluctuations and correlations

#109 : J. T. Mitchell
The low- to high-pT evolution of charged hadron azimuthal correlation functions: from HBT to jets

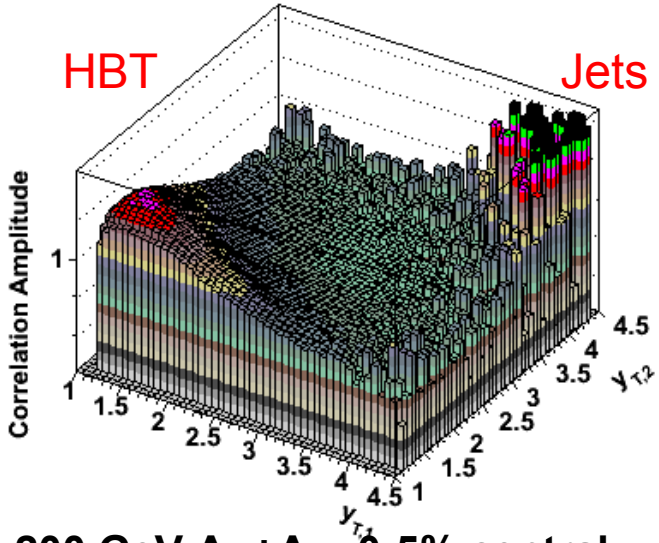
#110 : J. T. Mitchell
A survey of multiplicity fluctuations in PHENIX

#120 : M. J. Tannenbaum
How to measure specific heat using event-by-event average pT fluctuations

**J. T. Mitchell : poster #109**




**62 GeV Au+Au, 0-5% central**




**200 GeV Au+Au, 0-5% central**

PHENIX

- University of São Paulo, São Paulo, Brazil
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- China Institute of Atomic Energy (CIAE), Beijing, P. R. China
- Peking University, Beijing, P. R. China
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- Yonsei University, Seoul 120-749, Korea
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- Kurchatov Institute, Moscow, Russia
- PNPI, Petersburg Nuclear Physics Institute, Gatchina, Leningrad region, 188300, Russia
- Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Vorob'evy Gory, Moscow 119992, Russia
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***13 Countries; 62 Institutions; 550 Participants****

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- Abilene Christian University, Abilene, Texas, USA
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- Florida Institute of Technology, Melbourne, FL 32901, USA
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- University of Tennessee (UT), Knoxville, TN 37996, USA
- Vanderbilt University, Nashville, TN 37235, USA

***as of March 2005**

# Backup Slide

# Variables of statistical mechanics as order parameters

$$\Phi(h) = -\left(\frac{\partial G}{\partial h}\right)_h$$

$\Phi$ : extensive variables (G, V, S, N, $\chi \cdots$)

$h$ : intensive variables (T, P, $\cdots$)

$G$ : Gibbs free energy

First order susceptiblity :

$$\chi = -\left(\frac{\partial G}{\partial h}\right)_{h'}$$

voulume : $V = \left(\frac{\partial G}{\partial P}\right)_T$

entropy : $S = -\left(\frac{\partial G}{\partial T}\right)_P$

compressibility : $\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{T,S}$

Second order susceptibility :

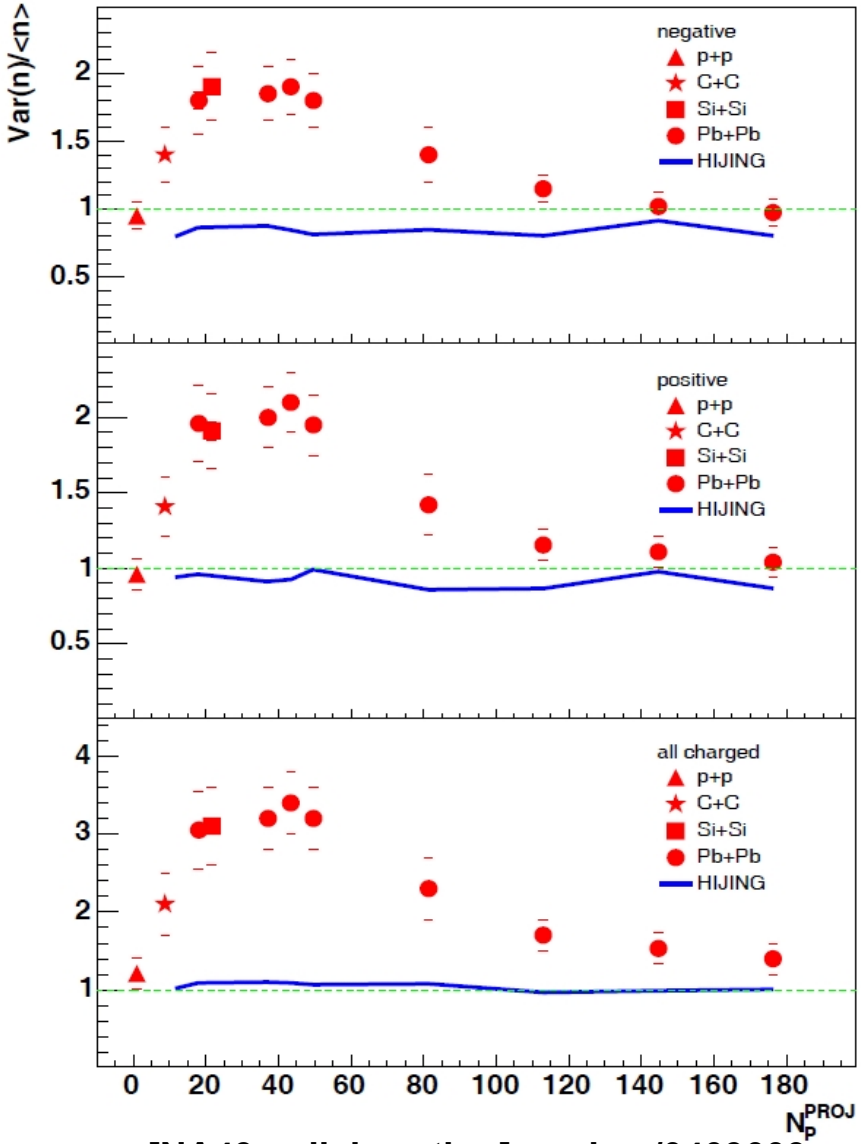
$$\chi = -\left(\frac{\partial^2 G}{\partial h^2}\right)_{h'}$$

specific heat : $C_h = -T\left(\frac{\partial^2 G}{\partial T^2}\right)_h$

correlation length : $\chi_k = \frac{\chi(T)}{1 + k^2 \xi^2}$

# Multiplicity fluctuations by variance

NA49: p+p, C+C, Si+Si, Pb+Pb 158 A GeV at SPS




[NA49 collaboration] nucl-ex/0409009

Variance of the multiplicity distribution is defined as;

$$< N >= \sum N \cdot P(N)$$

$$Var(N) = \sum (N - < N >)^2 P(N)$$

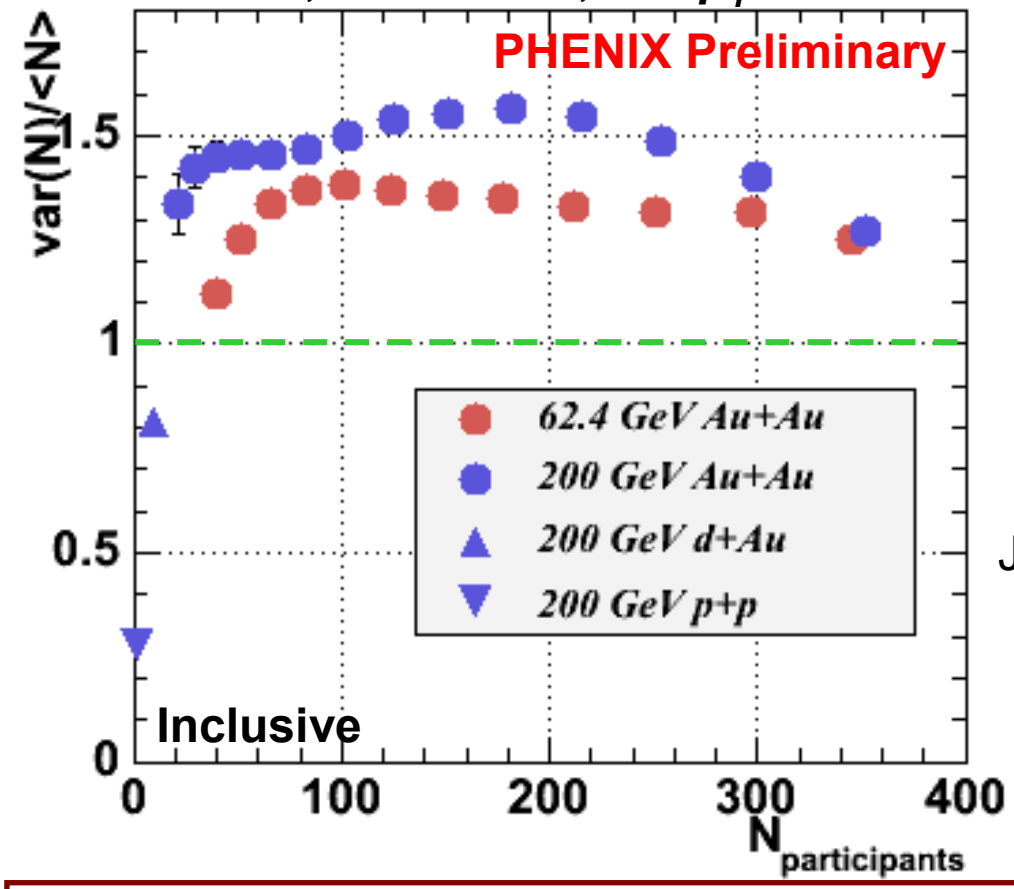
$$= < N^2 > - < N >^2$$

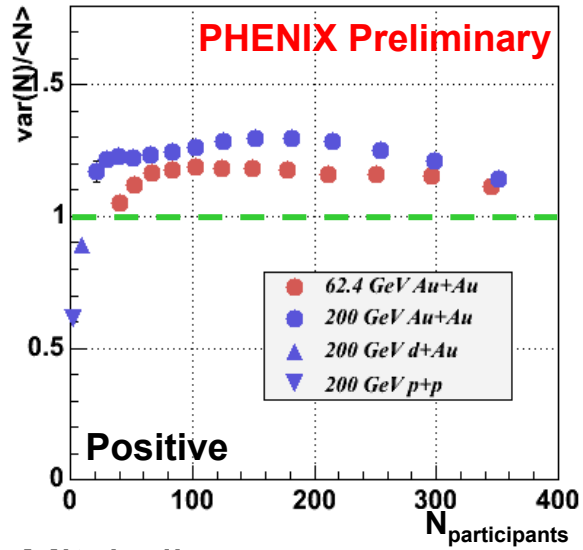
Normalized variance, *Var(N)/<N>,* is used as an observable of multiplicity fluctuation. In the case of Poisonnian distribution, the variance equals mean value and this observable indicates 1.

Deviations of multiplicity fluctuation from Poisonnian distribution was reported in the middle range of the number of projectile participant nucleon at SPS energy.

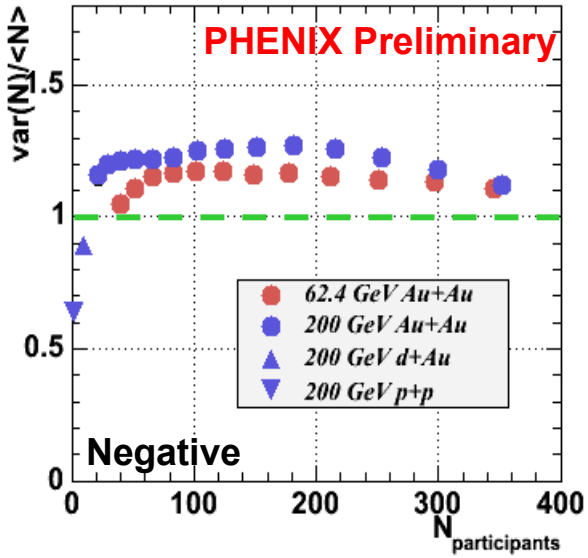
# Multiplicity fluctuations in PHENIX

$\Delta\eta$ <0.7, $\Delta\Phi < \pi$ rad, 0.2<$p_T$<2.0 GeV/c



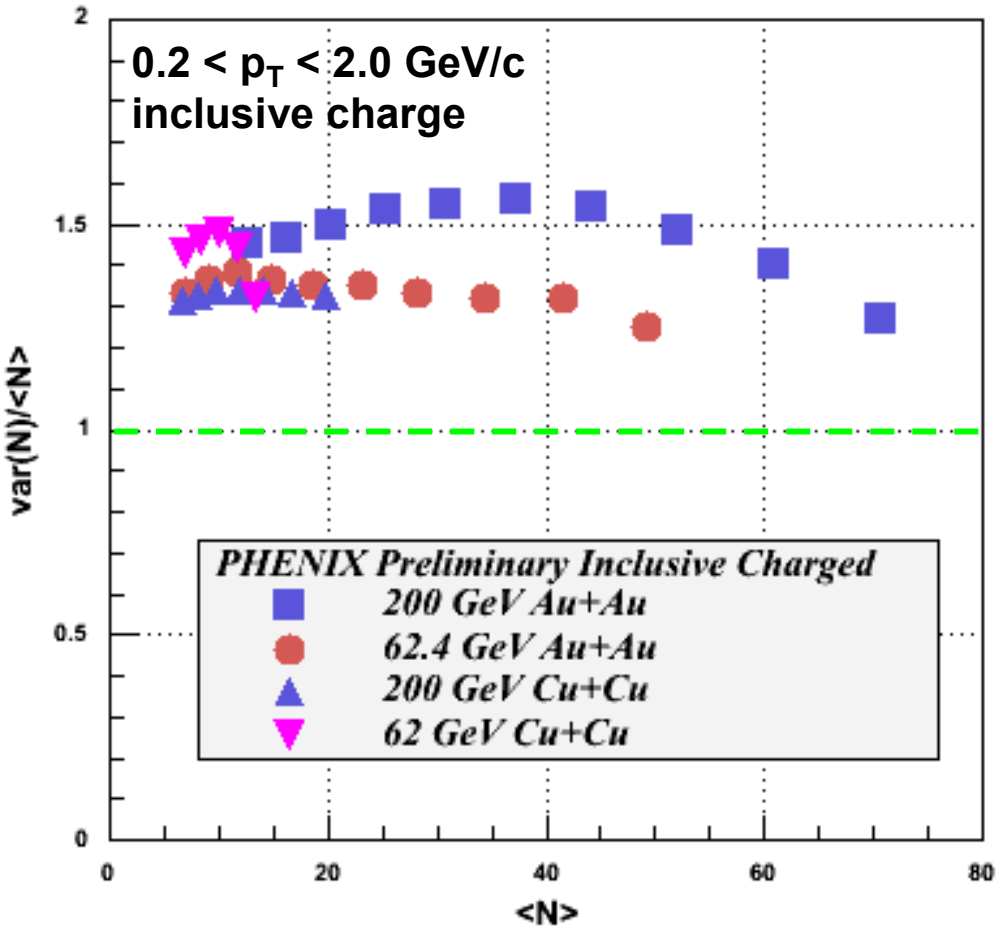



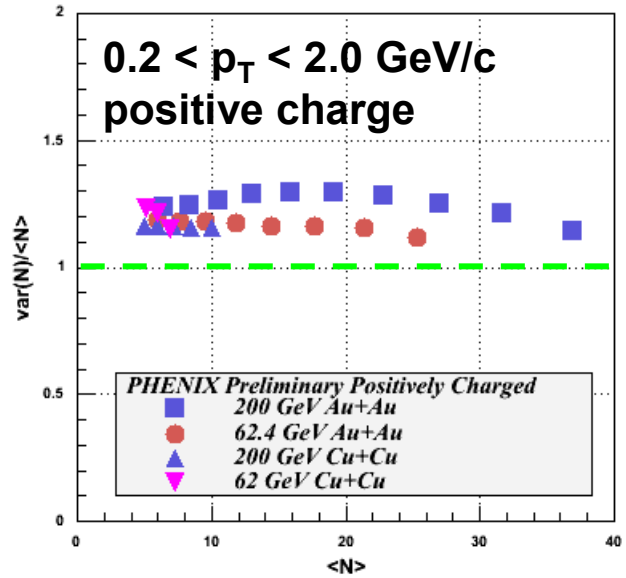

Jeffery T. Mitchell

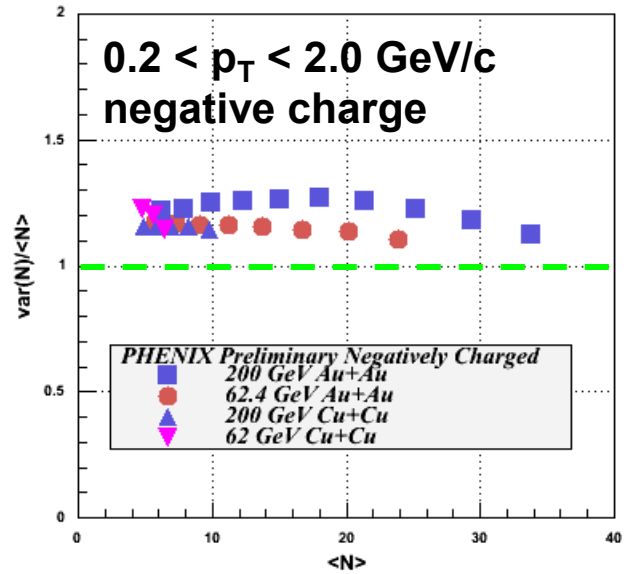



A different behavior of the multiplicity fluctuations as function of number of participants is observed at RHIC energies as compared to SPS. There are no difference about the multiplicity fluctuation between positive and negative charge.

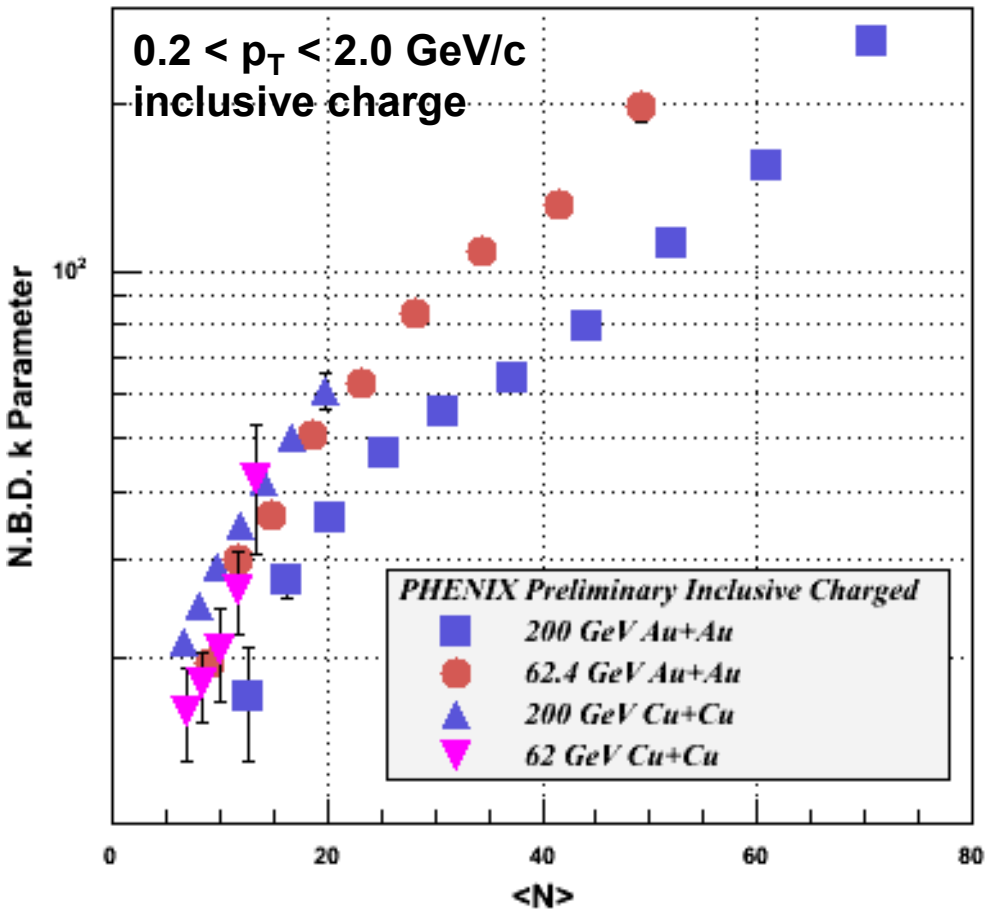
# Normalized variance as a function of average multiplicity

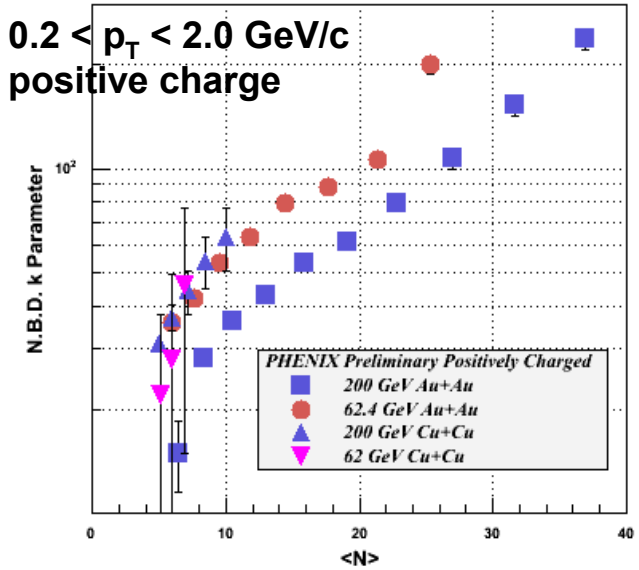


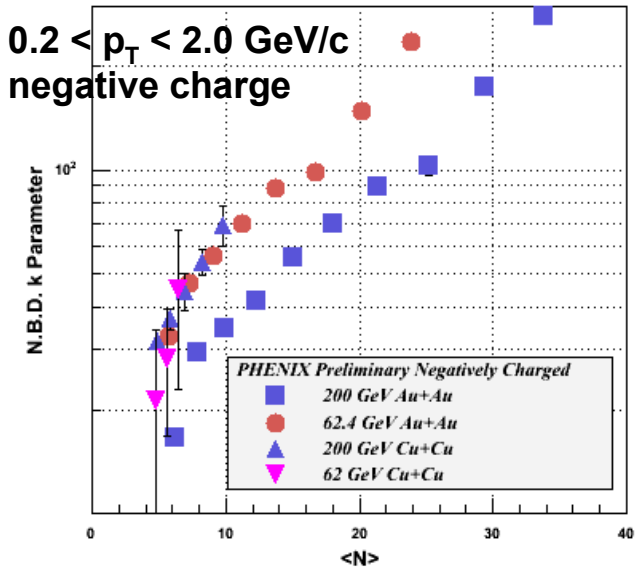



# NBD k parameters as a function of average multiplicity

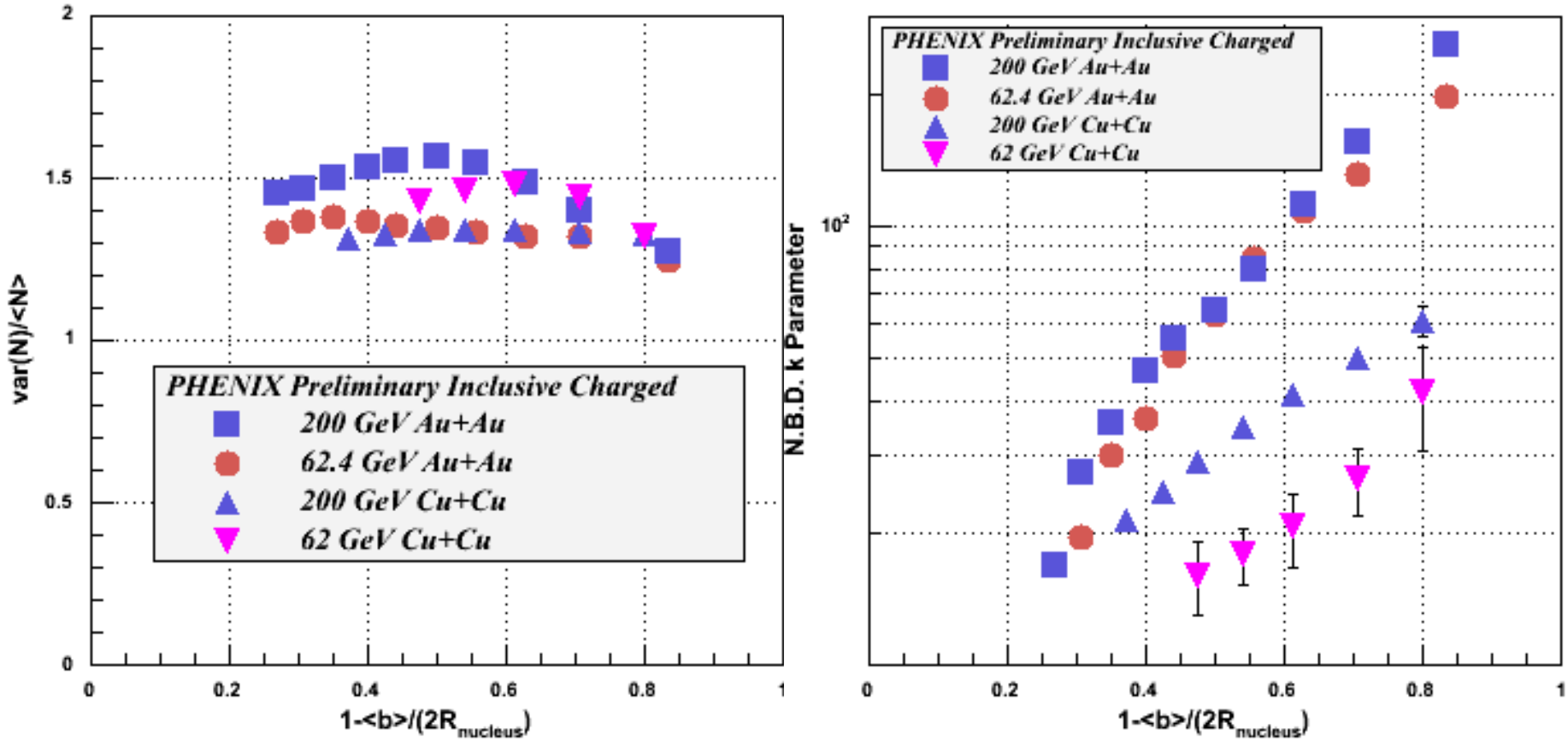


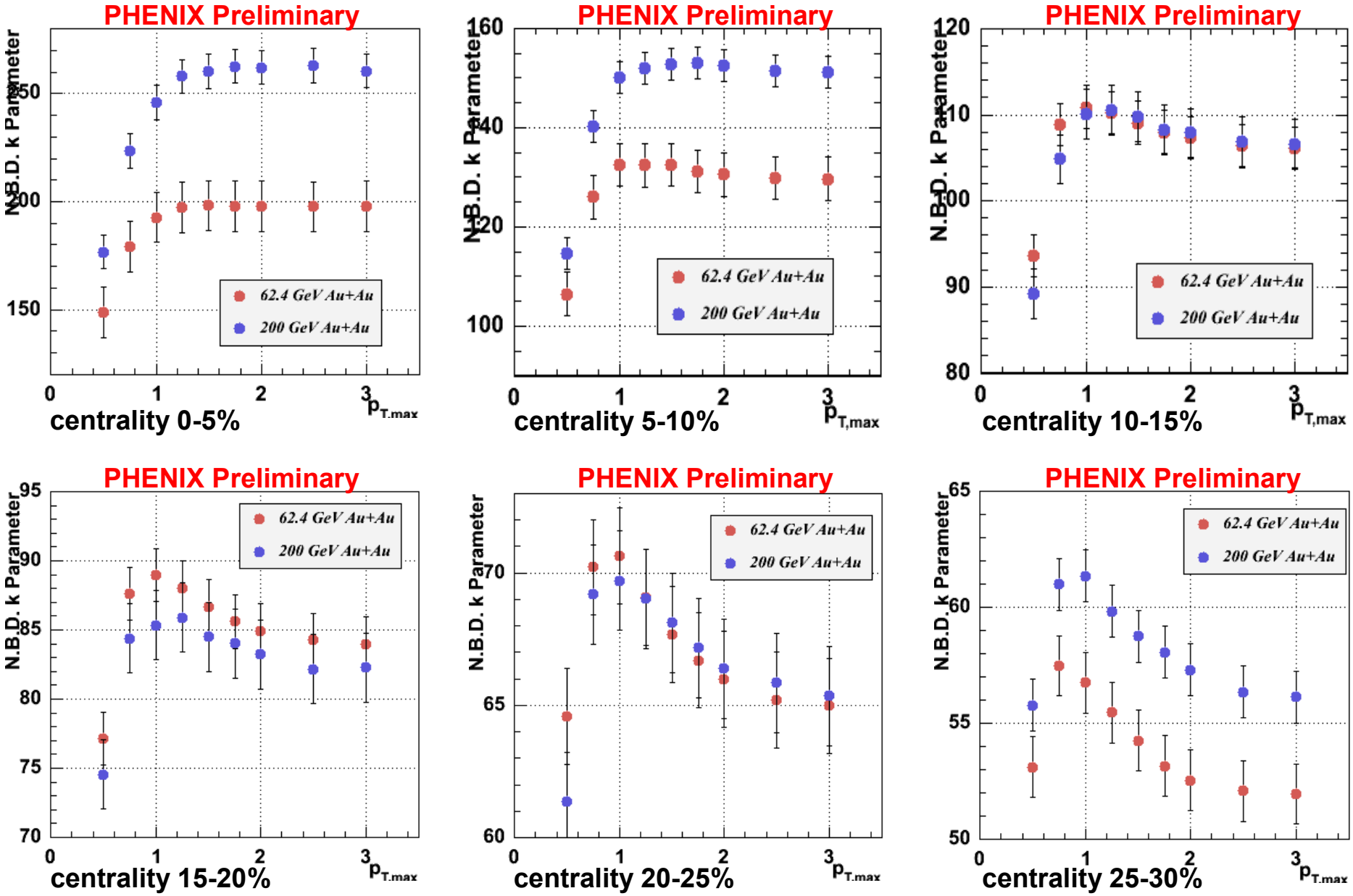



# Multiplicity fluctuations as a function of collision overlap geometry

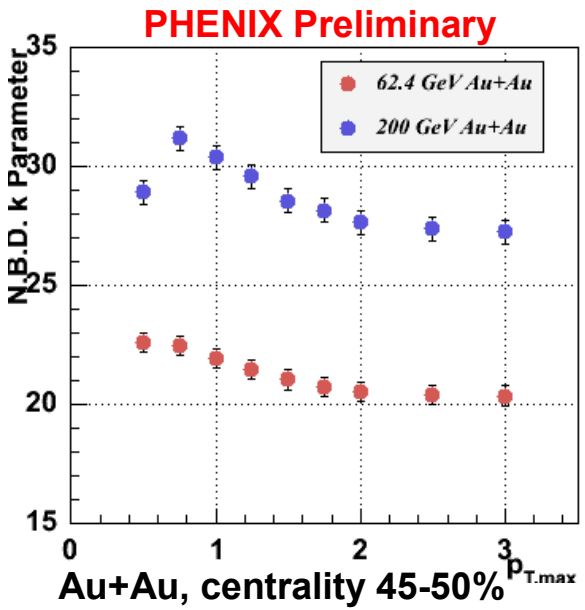
When plotted as a function of a measure of the collision overlap geometry (fractional impact parameter divided by the nuclear diameter - so a head-on collision = 1.0), the 62 GeV Cu+Cu fluctuations are less Poissonian.

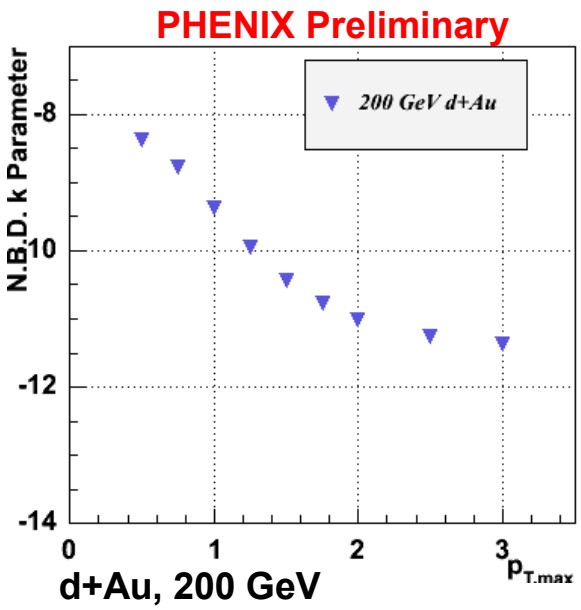


# $\delta p_T$ ($p_T$>0.2 GeV/c) dependence of NBD k



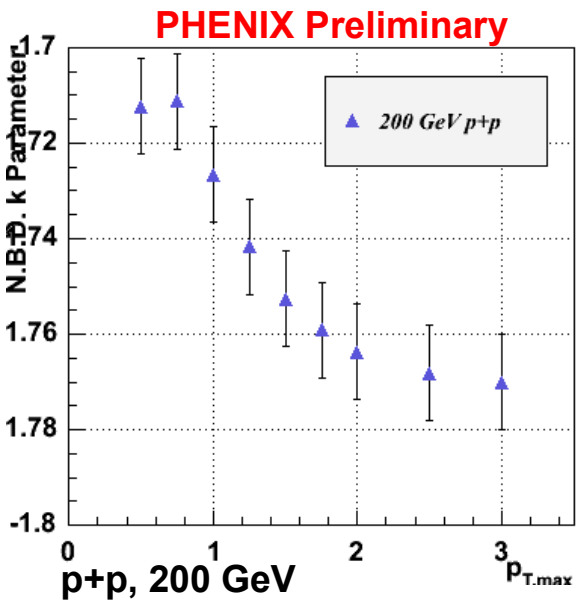
# Comparison for Au+Au, d+Au and p+p

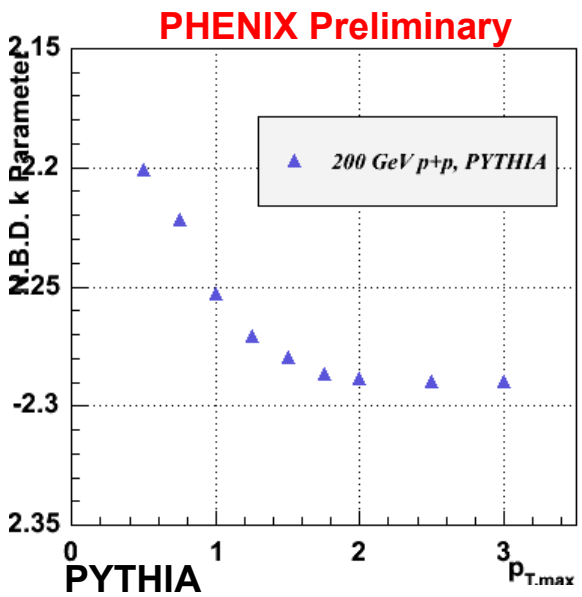



$\delta\, p_T$ dependence of NBD k parameters in Au+Au peripheral collisions are approaching toward the similar shape of d+Au and p+p with decreasing the centrality.

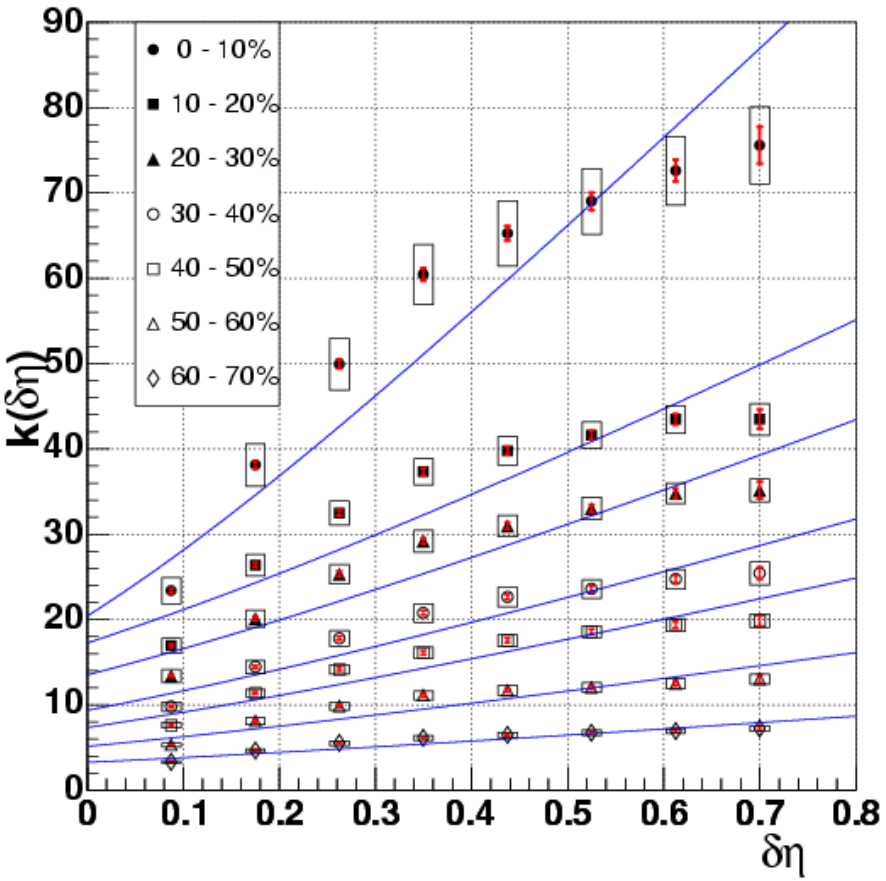
A behavior of NBD k parameters as a function of $\delta\, p_T$ at p+p collisions measured by PHENIX is agree with PYHTIA qualitatively.

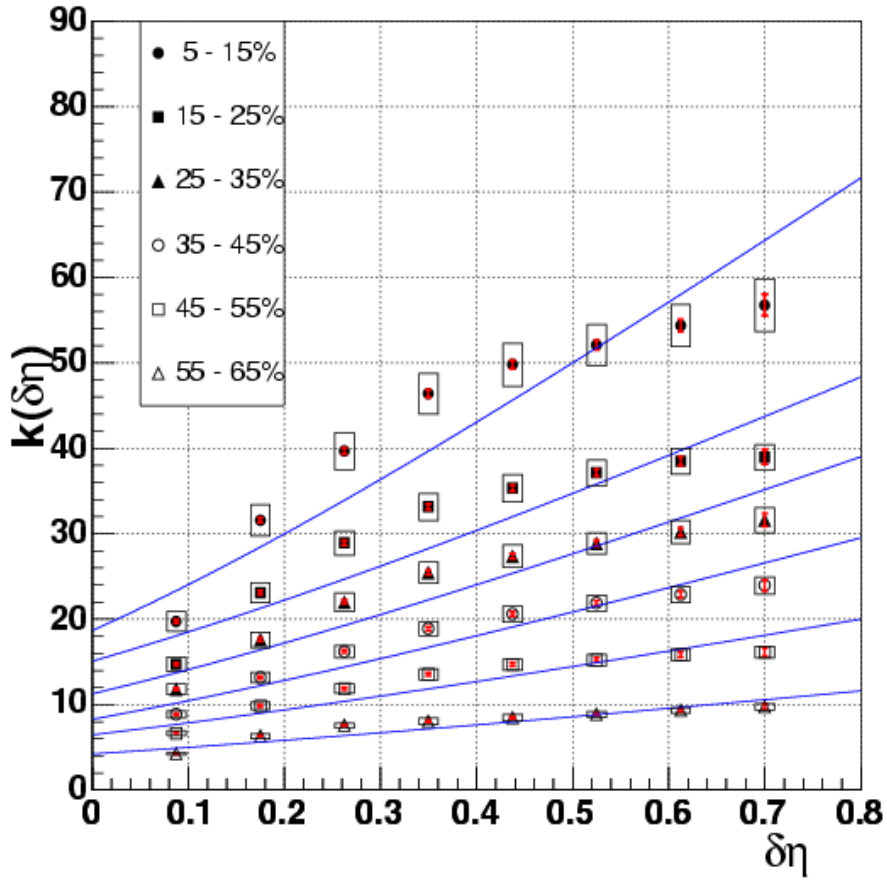


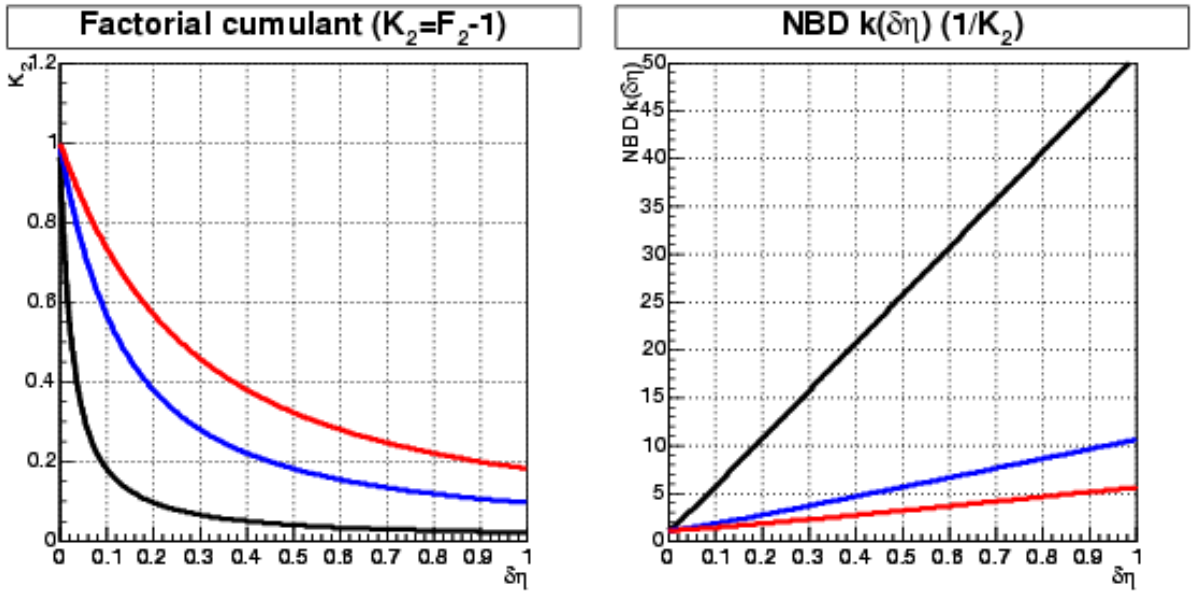
# Fit by the E802 type correlation function

- E802 type integrated two particle correlation function dose not agree with PHENIX data by taking all range of pseudo rapidity into account.

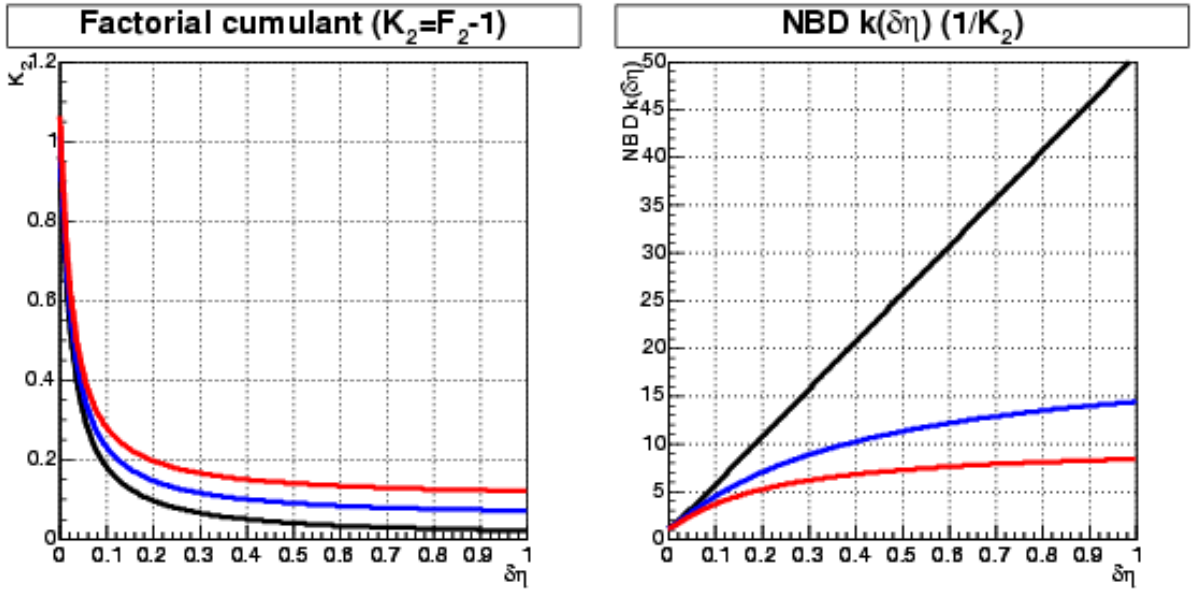


# Two component model



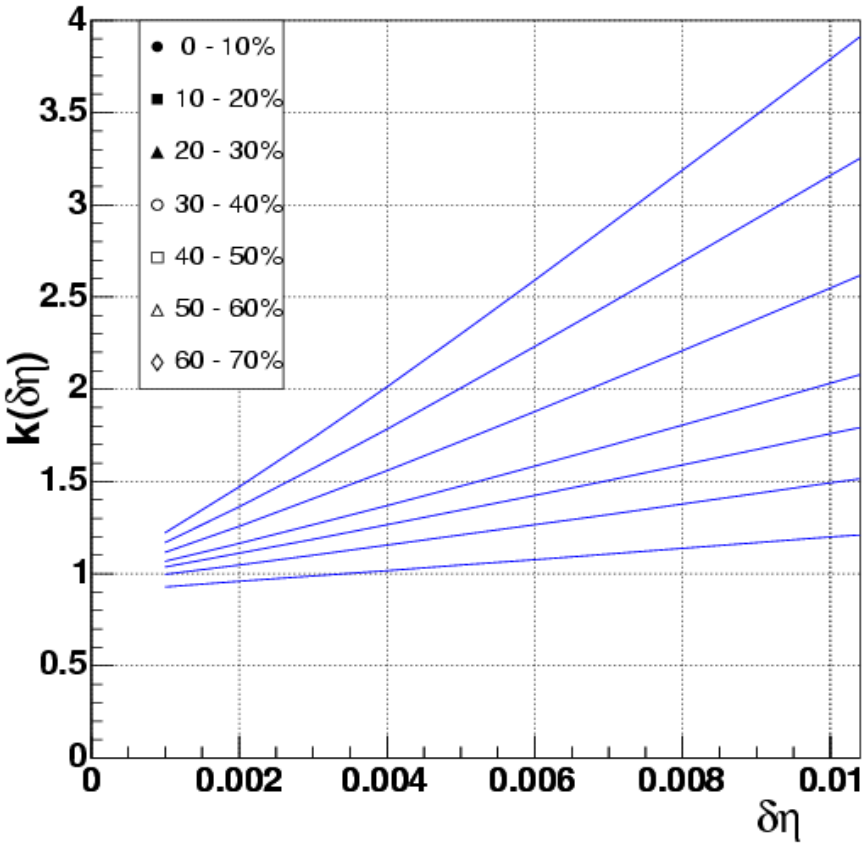

Black : b=0.0, $\xi$=0.01
Blue : b=0.0, $\xi$=0.05
Red : b=0.0, $\xi$=0.10

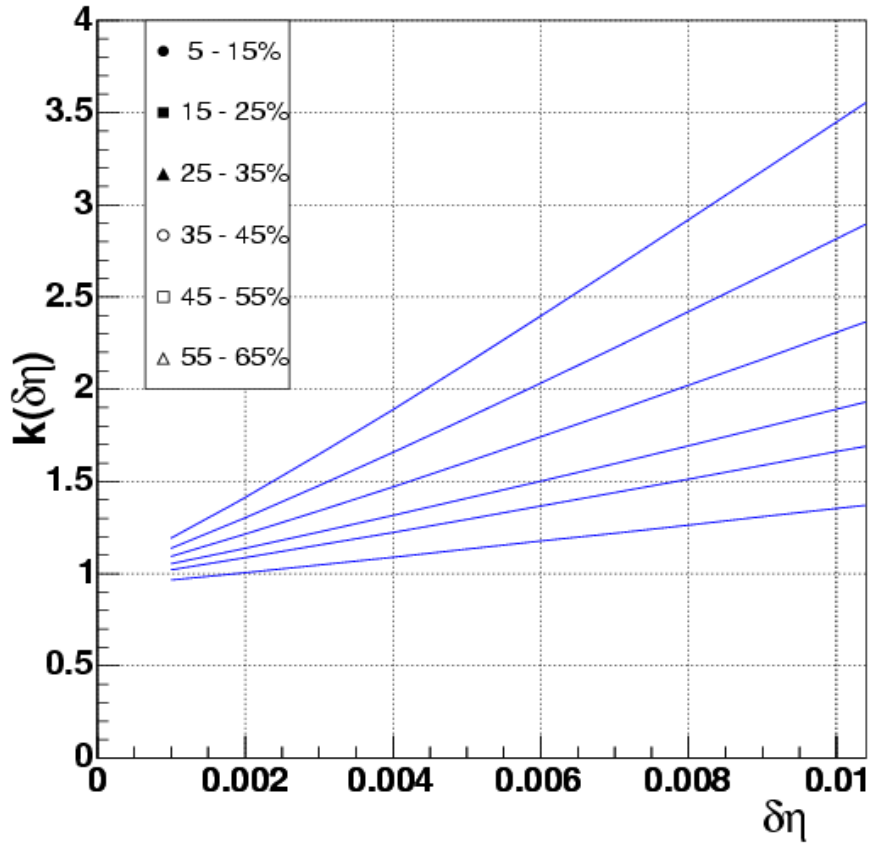



Black : b=0.0, $\xi$=0.01
Blue : b=0.1, $\xi$=0.01
Red : b=0.2, $\xi$=0.01

# Zoom up for small $\delta\eta$

- Lines are just extrapolated obtained fitting curve for small area ( $\delta\eta$ <0.01).
- K parameters converge finite value. Factorial moment/cumulant do not diverge.

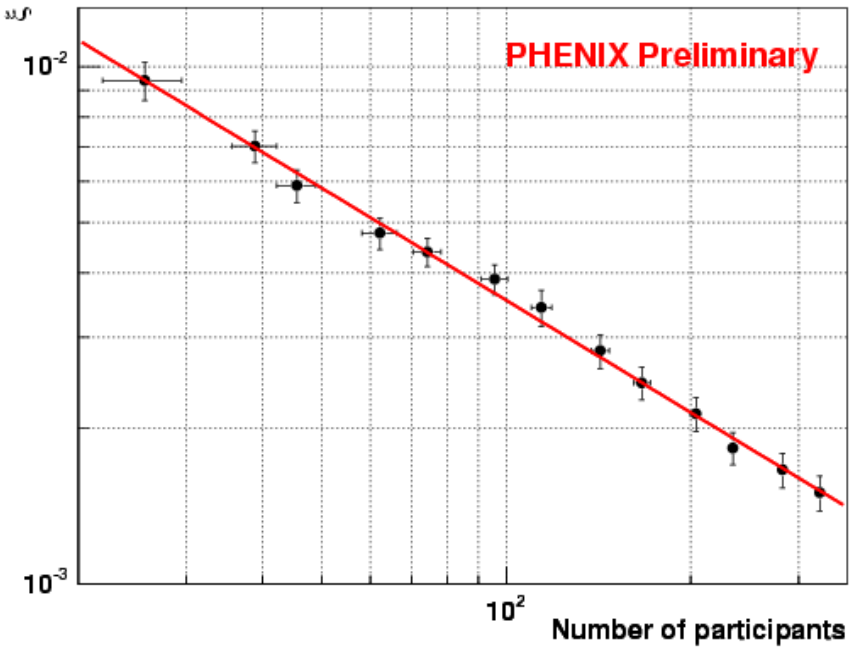


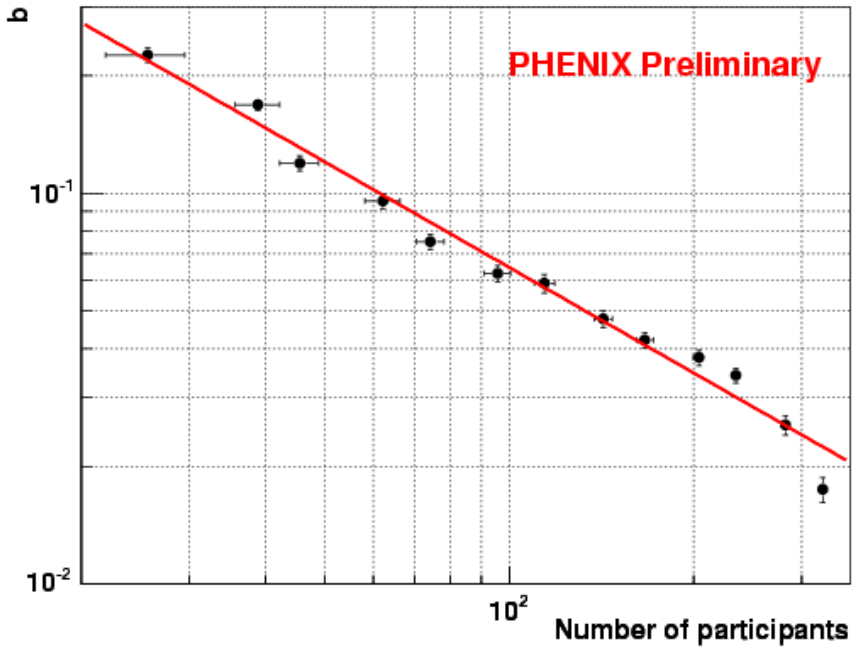



# Linearity in log-log plots

$$\xi \propto |T - T_c|^{\alpha} = \beta \, |N_{part}|^{\alpha}$$

$$\log(\xi) \propto \alpha \log(N_{part}) + \beta$$




$\alpha$ = -0.72 ± 0.032

$\beta$ = 0.097 ± 0.015

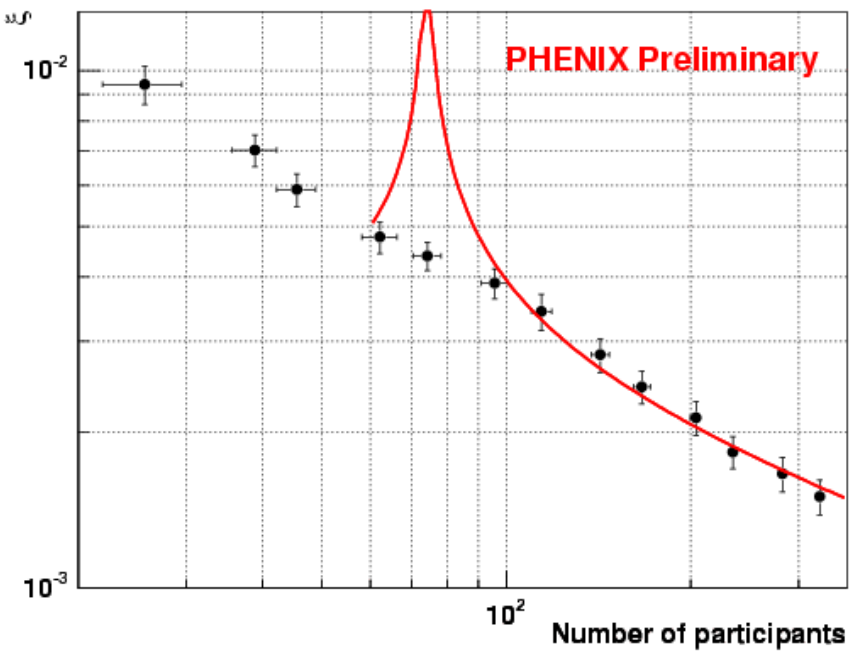
$\alpha$ = -0.90 ± 0.027

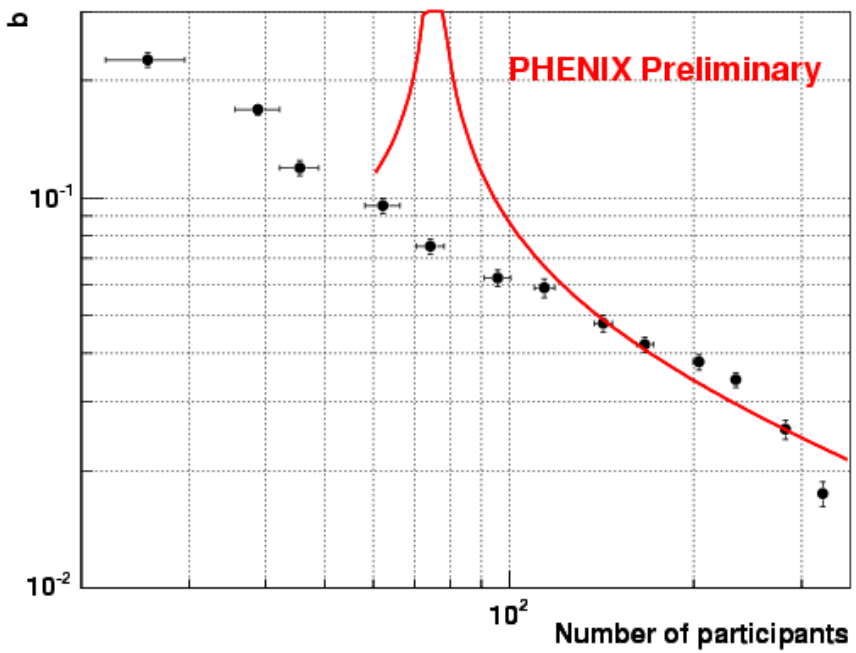
$\beta$ = 4.02 ± 0.540

# Divergence...!?

$$\xi \propto |T - T_c|^{\alpha} = \beta \, |N_{part} - N_{critical}|^{\alpha}$$

Fitting range : 60 < $N_{part}$ < 400

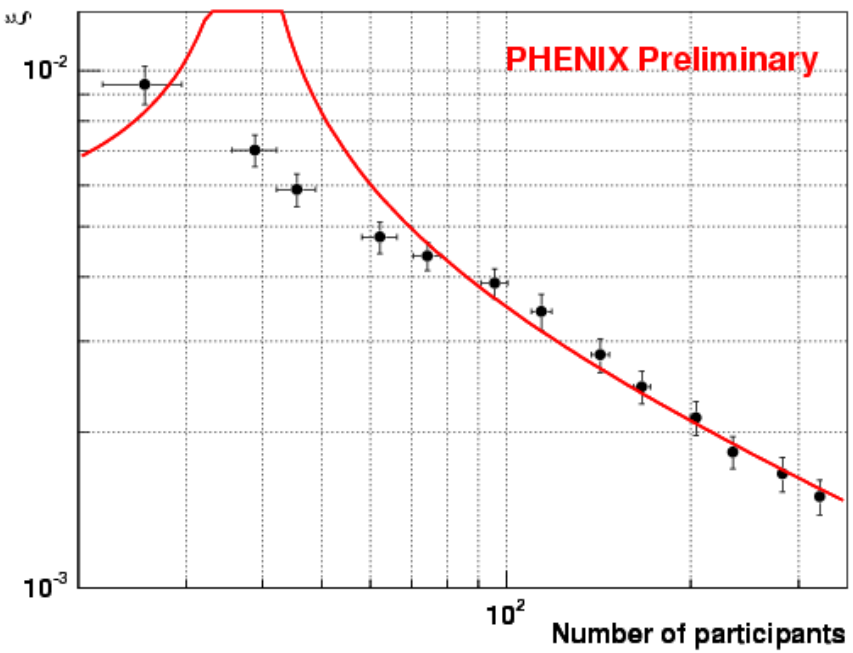


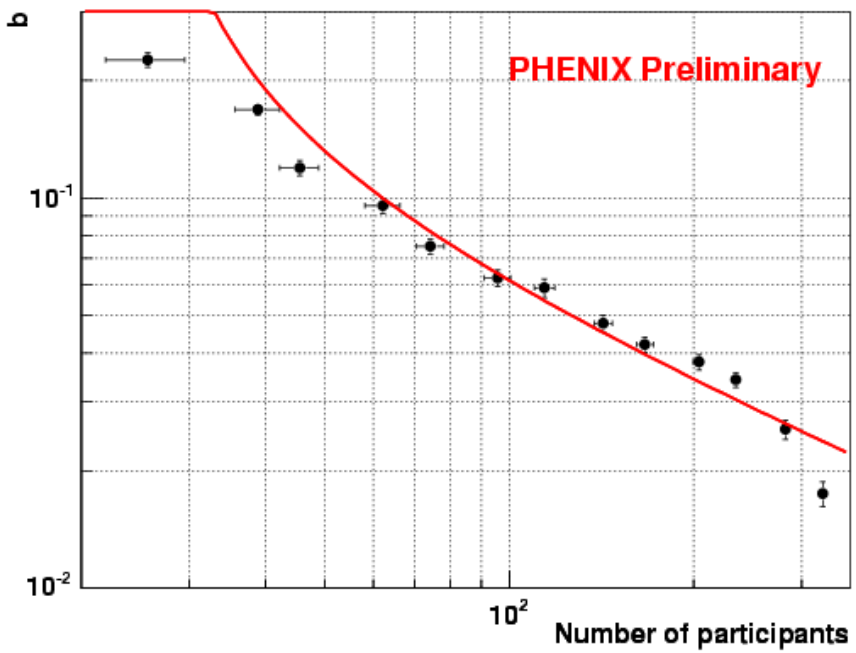



# Divergence…!?
# could not find critical points by only the fitting

$$\xi \propto |T - T_c|^{\alpha} = \beta \, |N_{part} - N_{critical}|^{\alpha}$$

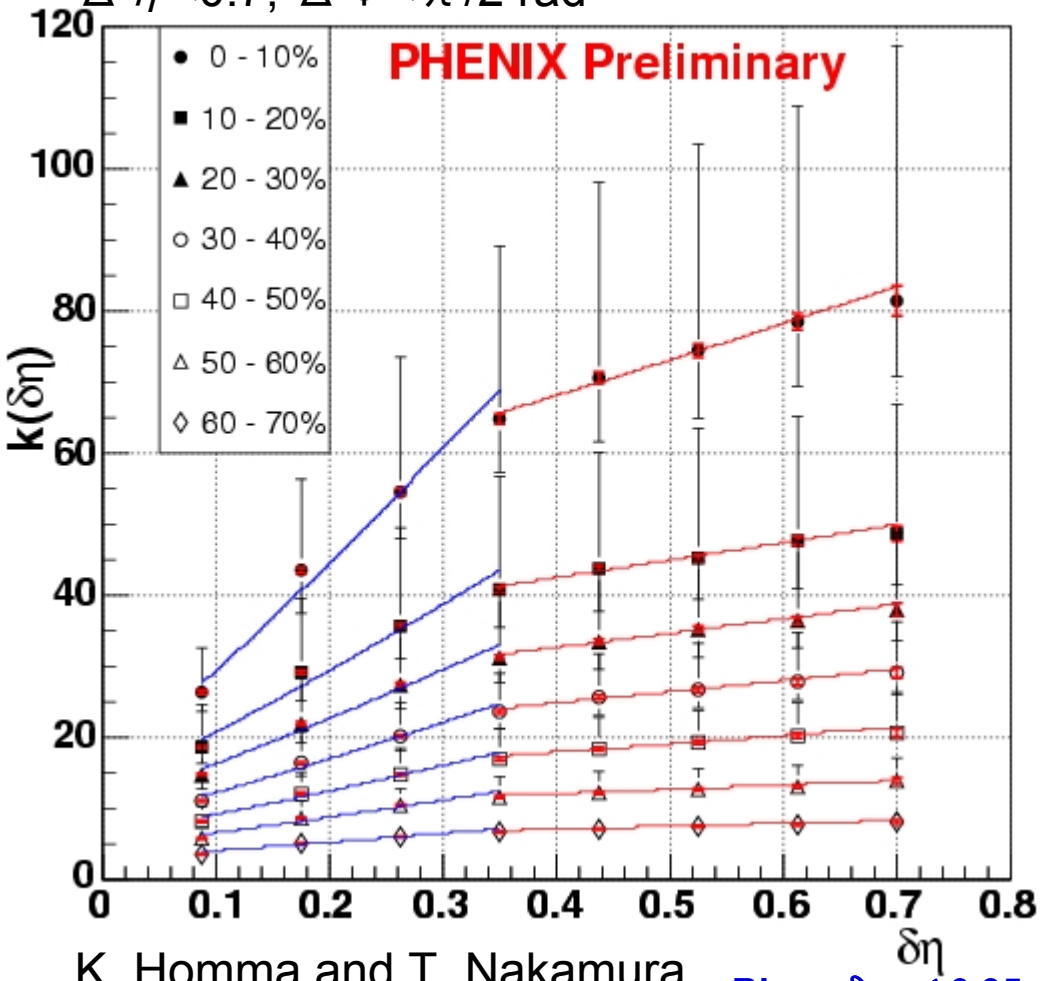
Fitting range : $0 < N_{part} < 400$



# $\delta\eta$ dependence of NBD k parameter

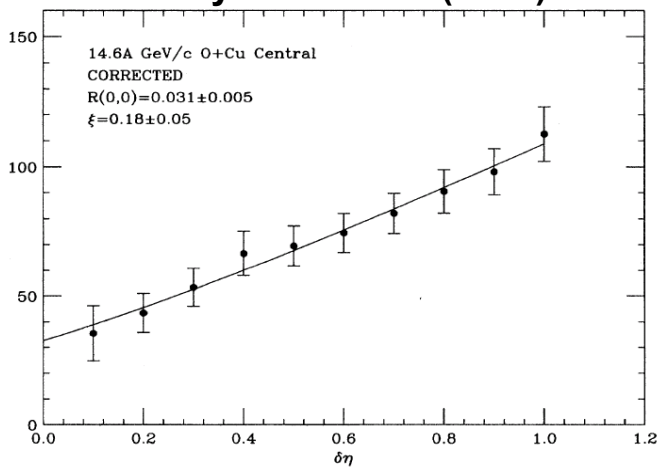
Au+Au 200 GeV, no magnetic field
$\Delta\eta$ <0.7, $\Delta\Phi < \pi/2$ rad




K. Homma and T. Nakamura

**Blue:** $\delta\eta \leq 0.35$
**Red :** $\delta\eta \geq 0.35$

**E802: Phys. Rev. C52 (1995) 2663**



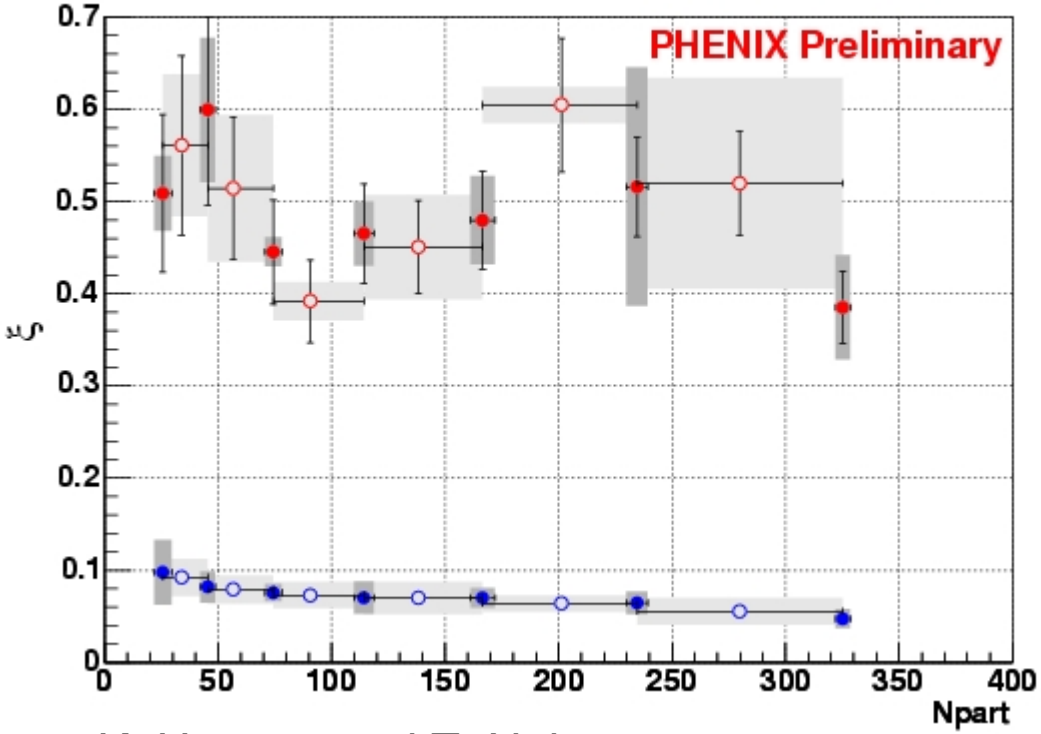

- Fitting funciton

$$k(\delta\eta) = \frac{1}{R_0}\frac{\delta\eta/2\xi}{[1-(\xi/\delta\eta)(1-e^{-\delta\eta/\xi})]}$$

NBD fit was performed for the different range of pseudo rapidity gap as shown in blue and red curve to extract the correlation length using the E802 type correlation function.

# Number of participants dependence of correlation length $\xi$

Au+Au 200 GeV, no magnetic field
$\Delta\eta < 0.7$, $\Delta\Phi < \pi/2$ rad




K. Homma and T. Nakamura

- **Fitting Range**

**Blue: $\delta\eta \leq 0.35$**
**Red : $\delta\eta \geq 0.35$**

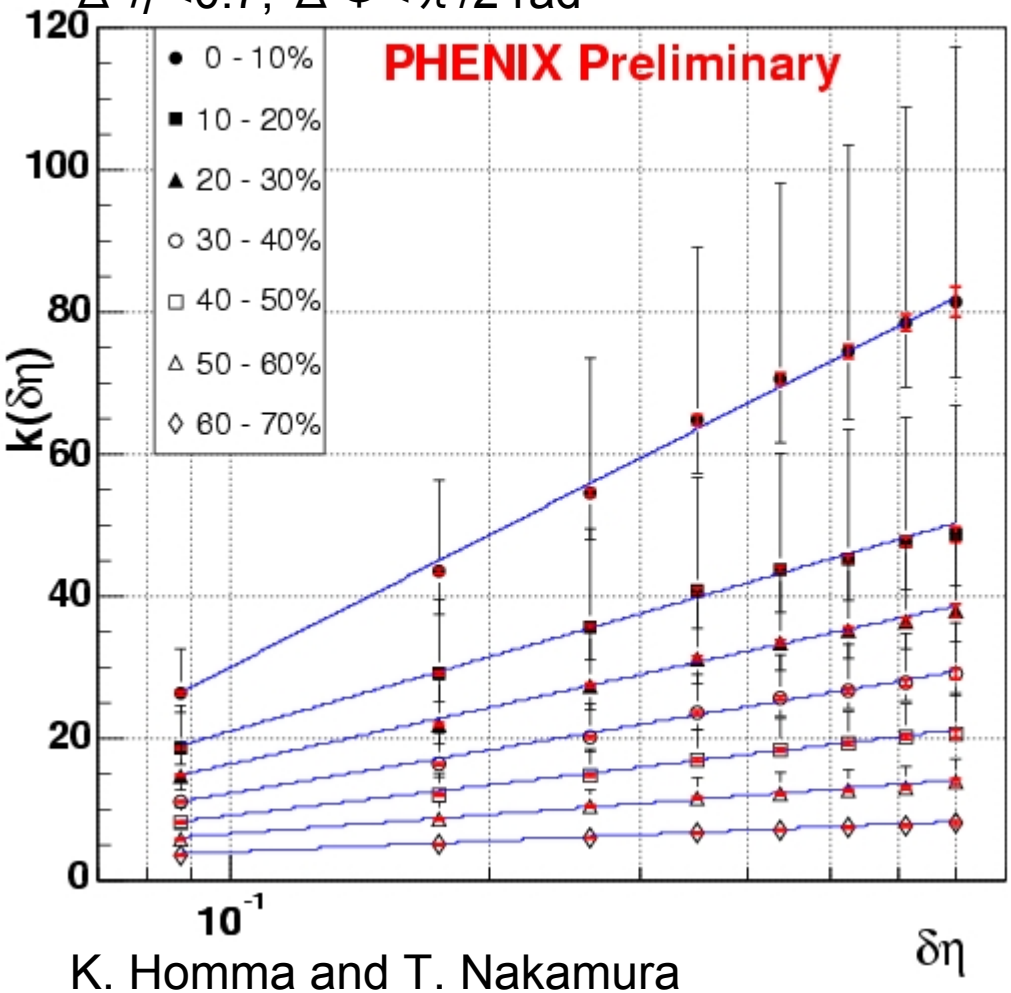
- **Centrality**

**filled circle : 0-70 % (10% interval)**
**open circle : 5-65 % (10% interval)**

Different behaviors about the extracted correlation length ( $\xi$ ) as a function of number of participants are observed in the different range of the pseudo rapidity gap. The correlation length at the range of large pseudo rapidity gap has a large fluctuation.

# Linear behavior of NBD k as a function of logarithmic $\delta\eta$

Au+Au 200 GeV, no magnetic field
$\Delta\eta < 0.7$, $\Delta\Phi < \pi/2$ rad




K. Homma and T. Nakamura

- **Fitting function**
  **$k(\delta\eta) = c1 + c2 \times \ln(\delta\eta)$**
  **c1, c2 : constant**
- **Fitting Range**
  **$0.09 \le \delta\eta \le 0.7$**

Relations of fluctuation and normalized factorial moments and fractal structure.

$$\frac{1}{k(\delta\eta)} + 1 = F_q(\delta\eta) \propto (\delta\eta)^{-\phi_q}$$

This power low relation of fluctuations and pseudo rapidity gap might suggest self similarity of correlation length!!...?

# Normalized factorial moment $F_q$

$$F_q(\delta) = \frac{< n(n-1)\cdots(n-q+1) >}{< n >^q}$$

$$F_2(\delta) = \frac{< n(n-1) >}{< n >^2} = \frac{< n >^2 - < n >}{< n >^2} = \frac{\sigma^2 + < n >^2 - < n >}{< n >^2}$$

$$= 1 + \frac{\sigma^2}{\mu^2} - \frac{1}{\mu}$$

$\mu \equiv < n >$ average multiplicity

$\sigma \equiv \sqrt{< n^2 > - < n >^2}$ standard deviation

$F_q(\delta) \propto (\delta)^{-\phi_q}$ relation between fractal (self-similarity) structures and normalized factorial moment

# NBD $k$ and factorial moment $F_q$

$$P_n^{(k)} = \frac{\Gamma(n+k)}{\Gamma(n-1)\Gamma(k)} \left( \frac{\mu / k}{1+\mu / k} \right)^n \frac{1}{(1+\mu / k)^k}$$

$$\sigma = \sqrt{\mu(1+\mu / k)}$$

$$\frac{\sigma^2}{\mu^2} = \frac{1}{\mu} + \frac{1}{k} \qquad F_2 - 1 = \frac{1}{k} \qquad F_2 = 1 + \frac{1}{k}$$

$$F_3 = \left(1+\frac{1}{k}\right)\left(1+\frac{2}{k}\right)$$

$$F_4 = \left(1+\frac{1}{k}\right)\left(1+\frac{2}{k}\right)\left(1+\frac{3}{k}\right)$$

$$F_q = F_{(q-1)}\left(1+\frac{q-1}{k}\right)$$

# Integral of correlation function and normalized factorial moments

$$\int^{\delta\eta} dy_1 \rho_1(y_1) =< n >$$

$$\int^{\delta\eta} dy_1 dy_2 \rho_2(y_1, y_2) =< n(n-1) >=< n >^2 F_2$$

$$\int^{\delta\eta} dy_1 \cdots dy_q \rho_q(y_1, \cdots y_q) =< n(n-1)\cdots(n-q+1) >=< n >^q F_q$$

$\rho_q(y_1, \cdots y_q)$ inclusive *q* particle density

when there are no correlation in rapidity

$$\rho_q(y_1, \cdots y_q) = \rho_1(y_1)\rho_2(y_2)\cdots\rho_q(y_q)$$

# Specific heat from average $p_T$ fluctuation

$$F_{p_T} = \left(\frac{\sigma_{M_{p_T}}}{\mu} - \frac{1}{\sqrt{n}}\frac{\sigma_{p_T}}{\mu}\right) / \frac{1}{\sqrt{n}}\frac{\sigma_{p_T}}{\mu}$$

$$\frac{\Delta\sigma^2}{\sigma^2} = 2\frac{\Delta\sigma}{\sigma} = 2F$$

$$\frac{\sigma^2_{M_{p_T}}}{\mu^2} - \frac{1}{n}\frac{\sigma^2_{p_T}}{\mu^2} = \left(1 - \frac{1}{n}\right)\frac{\sigma^2_T}{\langle T\rangle^2}$$

**Korus, et al, PRC 64, (2001) 054908**

$$1/C_V = \sigma^2_T / \langle T\rangle^2$$

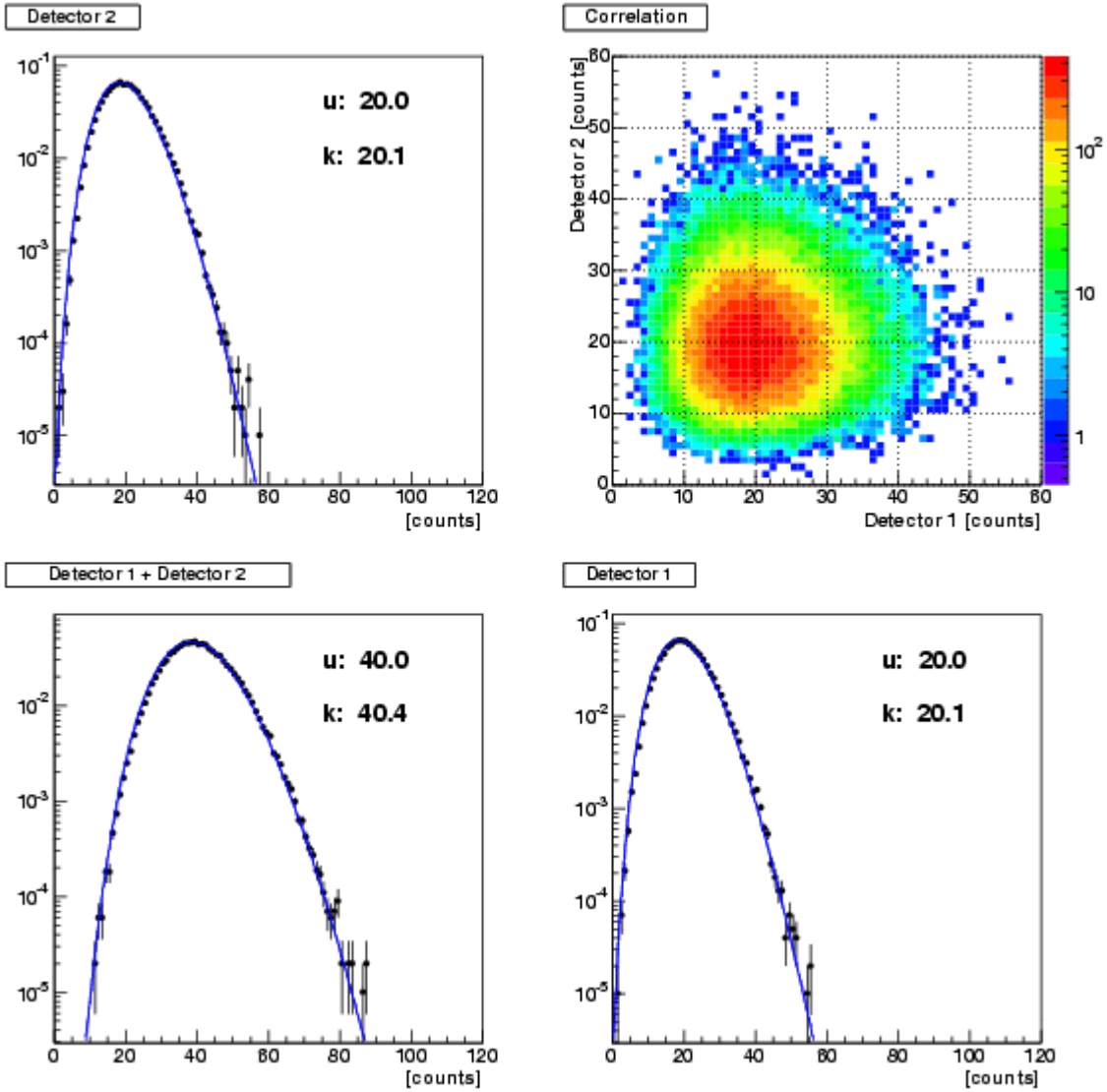
$$c_v = C_V / \langle N_{tot}\rangle$$

$$c_v = \frac{\langle n\rangle}{\langle N_{tot}\rangle}\frac{1}{F_{p_T}}$$

M. J. Tannenbaum,
2nd International Workshop on the Critical Point and Onset of Deconfinement

*n* represents the measured particles while $N_{tot}$ is all the particles, so $n/N_{tot}$ is a simple geometrical factor for all experiments

# Random particle emission pattern based on NBD

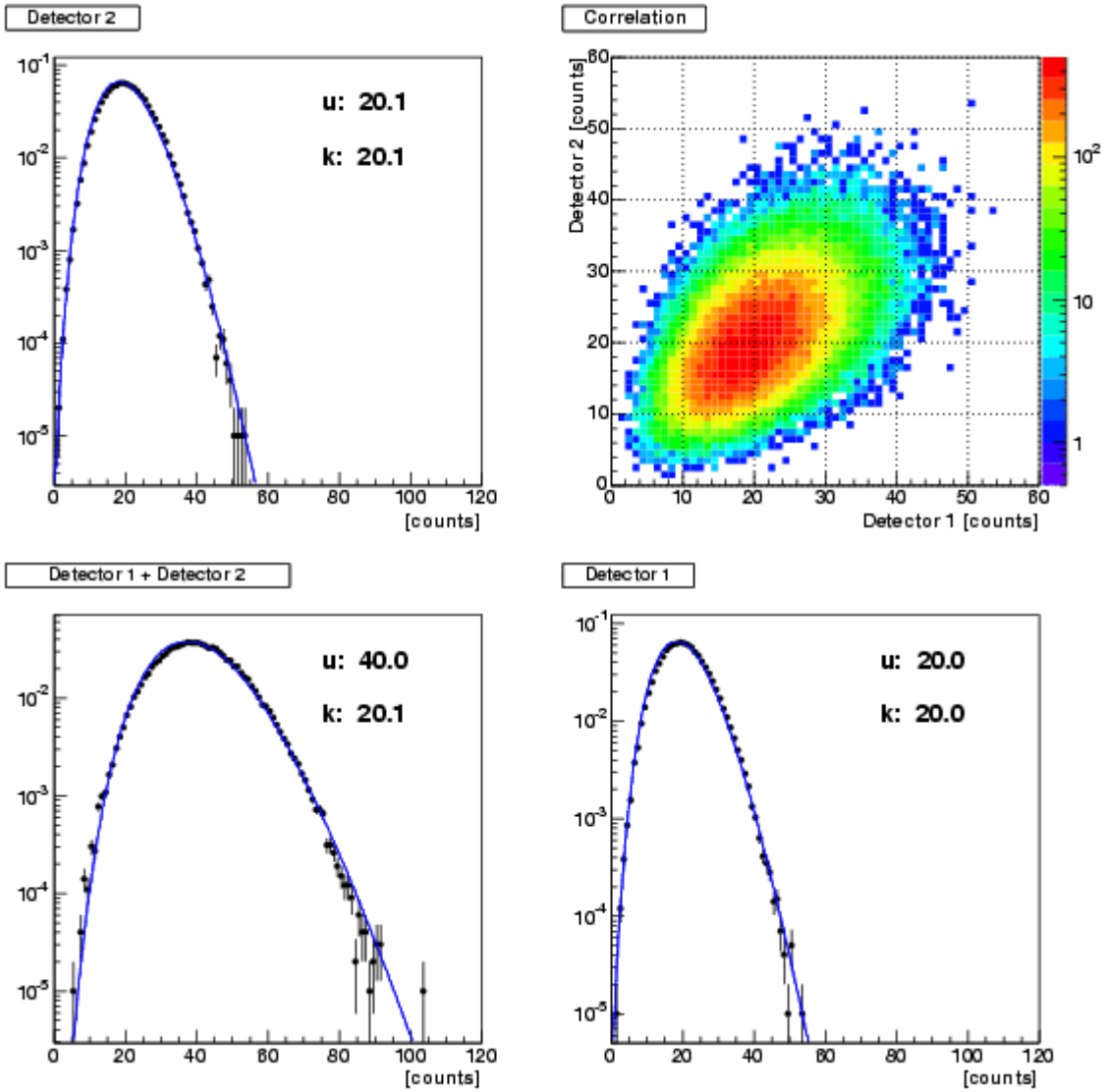



| Detector 1 | Detector 2 |
|---|---|

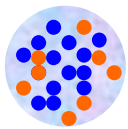
Emission source

In the case of there are no correlation about the particle emission, the value of NBD k parameters are summed up.

# Correlated particle emission pattern into the phase-space




| Detector 1 | Detector 2 |
|---|---|



Emission source

If there are correlations, NBD k parameters do not increase according to the size of detector acceptance.

# Correlation functions and correlation length

**Used in E802**

$$C_2 = 1 + R(0,0)e^{-|y_1 - y_2|/\xi}$$

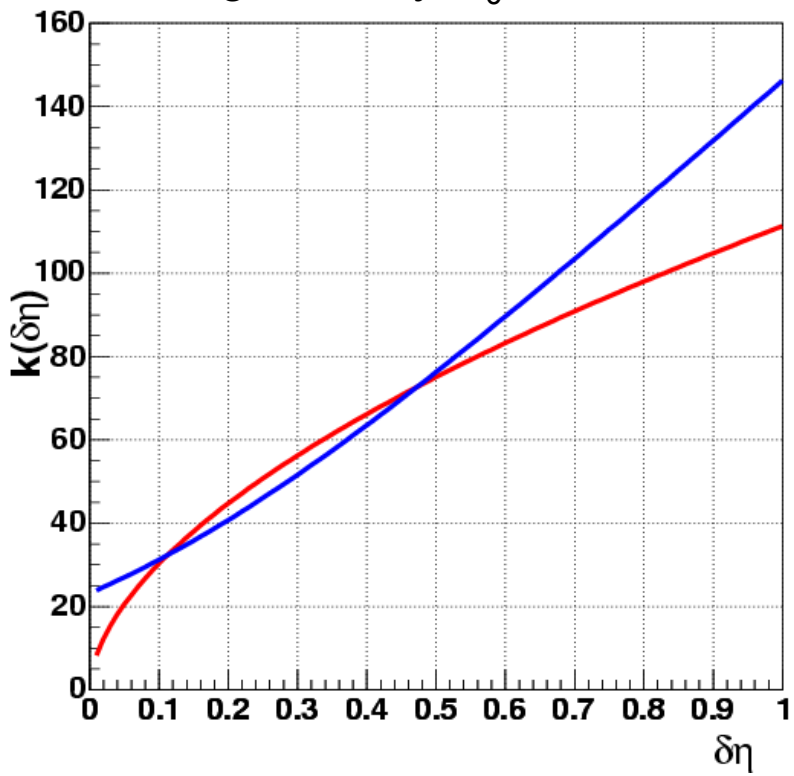
$$k(\delta\eta) = \frac{1}{R_0}\frac{\delta\eta/2\xi}{[1-(\xi/\delta\eta)(1-e^{-\delta\eta/\xi})]}$$

**General correlation function**

$$C_2 = 1 + \frac{R_0}{|y_1 - y_2|^{\alpha}} e^{-|y_1 - y_2|/\xi}$$

$$k(\delta\eta) = \frac{\delta\eta}{\int^{\delta\eta} \frac{R_0}{y^{\alpha}} e^{-y/\xi} dy}$$

Using arbitrary $R_0$, $\xi$ and $\alpha$.

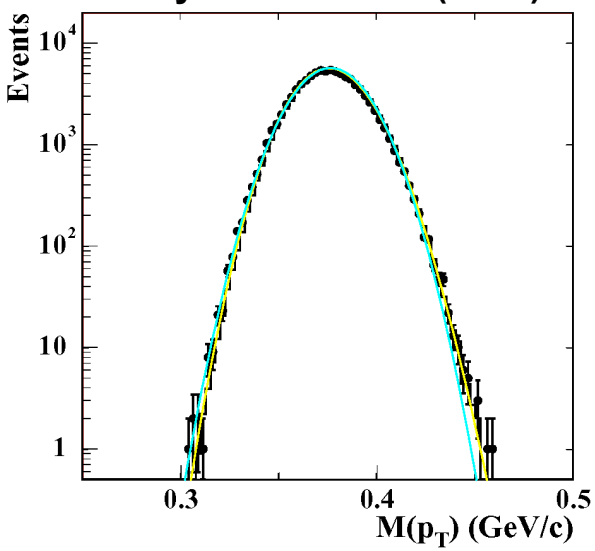



One may discuss an effective potential form of a deconfined field when assume the correlation functions.

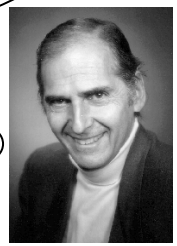
$\xi$ : correlation length, $\alpha$ : critical exponent

# Average $p_T$ fluctuation
# Published by Phys. Rev. Lett. 93, 092301 (2004)

NA49: Phys. Lett. B 459 (1999) 679

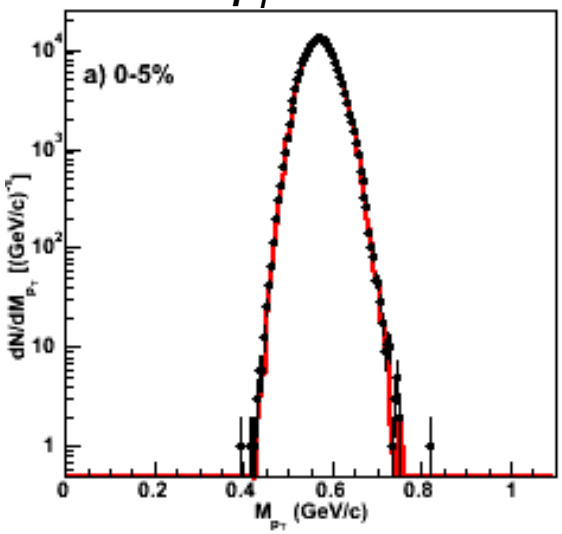



**Already, famous!**
It's not a Gaussian…
it's a Gamma distribution!



M.J. Tannenbaum,
Phys. Lett. B 498 (2001) 29

PHENIX: Au+Au 200GeV
$0.2 < p_T < 2.0$ GeV/c

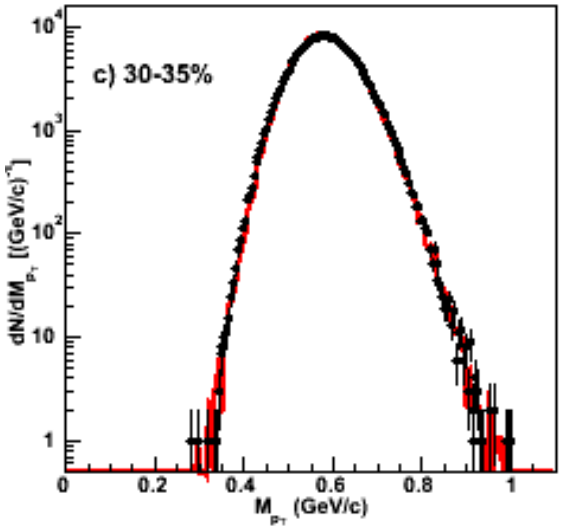



Magnitude of average $p_T$ fluctuation

$$\omega_{p_T} = \frac{(<M_{p_T}^2> - <M_{p_T}>^2)^{1/2}}{<M_{p_T}>} = \frac{\sigma_{M_{pT}}}{<M_{p_T}>}$$

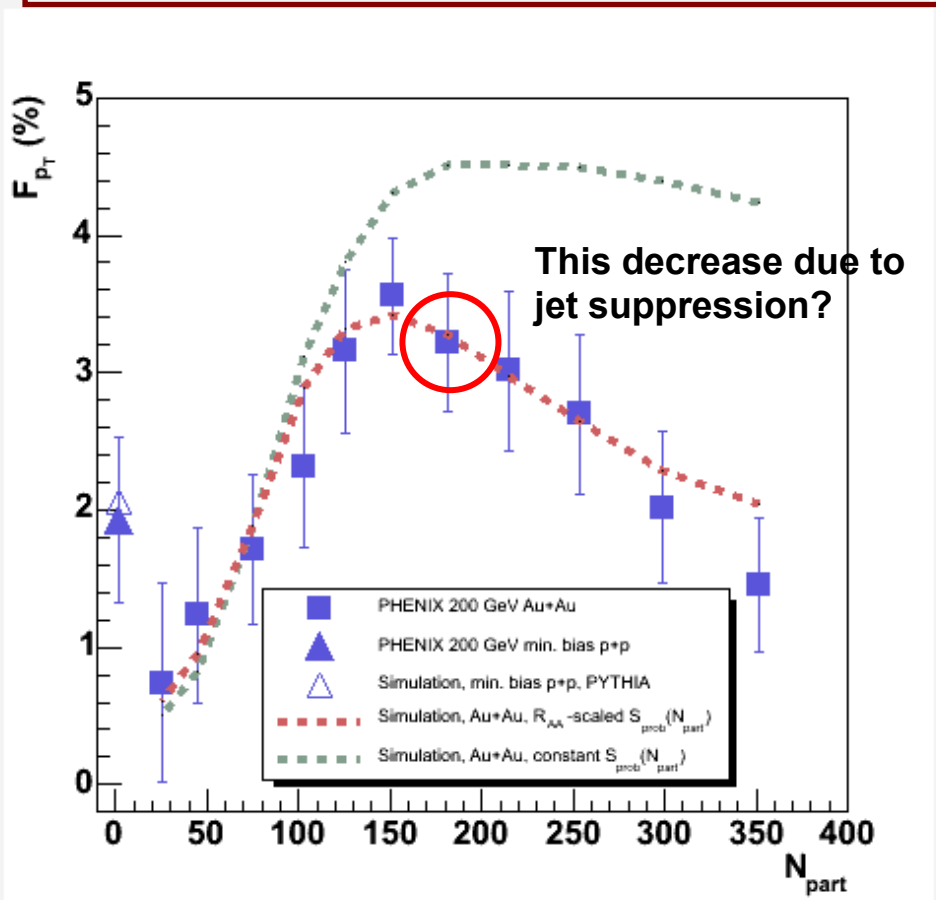
Fractional deviation from mixed events

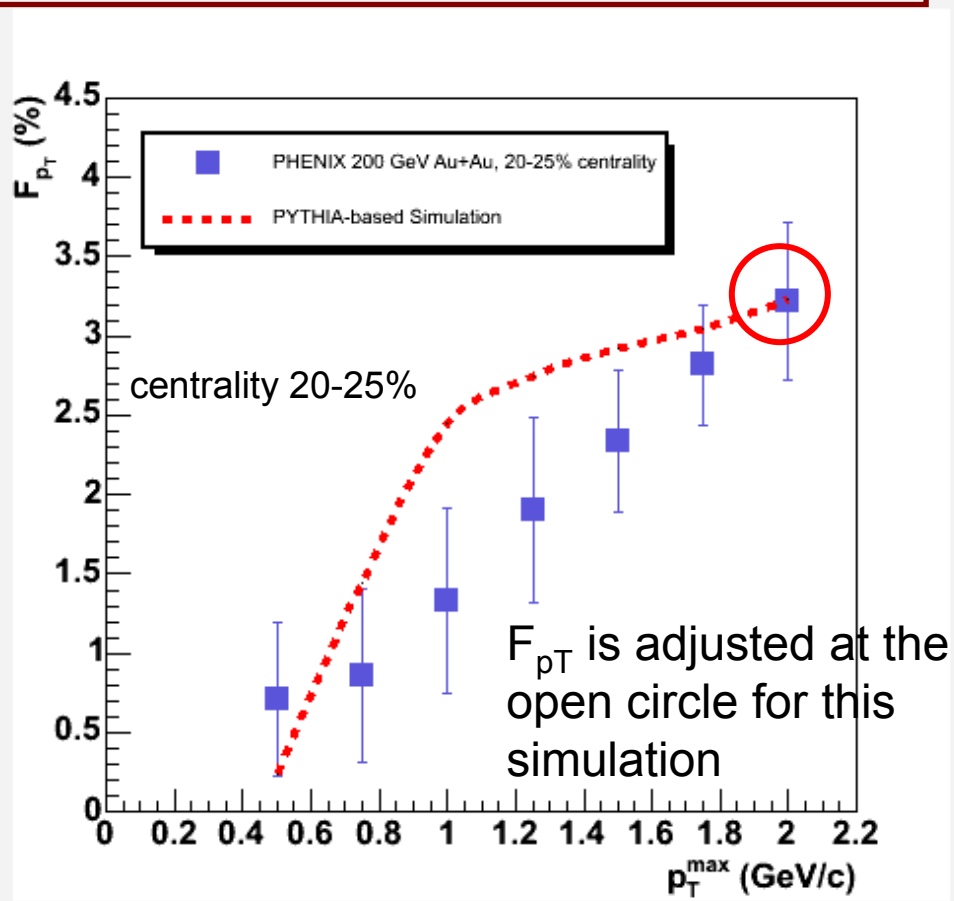
$$F_{p_T} = \frac{[\omega_{(p_T,data)} - \omega_{(p_T,mixed)}]}{\omega_{(p_T,mixed)}}$$



# Contribution of Jet/Jet suppression to the average $p_T$ fluctuation

PYTHIA based simulation, which contains scaled hard-scattering probability factor ($S_{prob}$) by the nuclear modification factor ($R_{AA}$), well agree with the measured $F_{pT}$. It might be indicate that jet suppression might contribute to the average $p_T$ fluctuation.



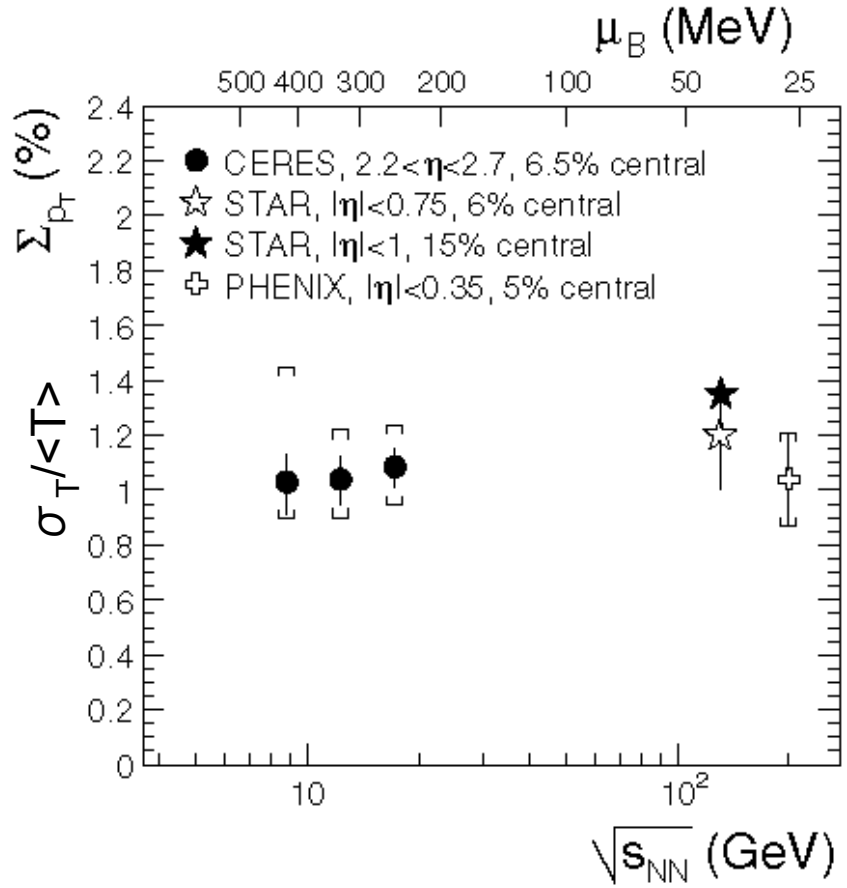
# Estimation of the magnitude of residual temperature fluctuations

$$\frac{\sigma_T}{<T>} = \sqrt{\frac{2F_{p_T}}{p(<N>-1)}}$$

$p \rightarrow$ inclusive $p_T$

**R. Korus and S. Mrowczynski, Phys. Rev. C64 (2001) 054908.**

| Experiment | $\sqrt{s_{NN}}$ | $\sigma_T/<T>$ (most central) |
|---|---|---|
| PHENIX | 200 | 1.8% |
| STAR | 130 | 1.7% |
| CERES | 17 | 1.3% |
| NA49 | 17 | 0.7% |




**H.Sako, et al, JPG 30, S (2004) 1371**

A signal of phase transition dose not emerge in the temperature fluctuation?